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Fast particles in drift wave turbulence

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ABSTRACT

This study aims to incorporate the effects of fast particles into our present fluid model for tokamak transport. The parameter $\varepsilon_f = \omega/\omega_f$, where ω is the mode frequency and ω_f is the typical frequency of the fast particles, which enters as a factor in front of the fast particle response. Thus, for trapped fast particles, where $\omega_f = \omega_{pres}$ the precession frequency of the fast particles, this parameter is of order 10^{-2} for drift waves, and thus, the fast particle response can be neglected. However, ε_f will be of order 1 for fast particle modes such as in the fishbone instability. An important turbulence property, affecting both these limits, is resonance broadening. Effects of resonance broadening have recently been considered for fast particle instabilities, often coupled directly to the linear growth rate, while we here consider the original Dupree formulation where the turbulence directly drives a nonlinear frequency shift. Resonance broadening has a general tendency to counteract dissipative wave particle resonances. This has been observed for fast particle instabilities. Here, there is a resonant external source for the fast particles, so the instability survives if this source is dominant over the resonance broadening. For drift waves, however, external sources are not resonant since $\varepsilon_f \ll 1$. Thus, the resonance broadening is able to remove the dissipative wave particle resonance completely.

I. INTRODUCTION

We will here discuss fast particle instabilities from the point of view of how these arise in kinetic and fluid models.[1–4] We here need to make use of several works on the basic background.[5–15] The first effects noted experimentally from fast particles were a stabilizing effect due to dilution.[15] The reason for this was that we must have quasi neutrality so that a population of fast ions meant that the number of background ions, driving various instabilities, like, e.g., the Ion Temperature Gradient (ITG) instability, had to be reduced. This effect is, of course, present also for impurities.

However, in 1983, fishbone instability was observed on the Poloidal Divertor Experiment (PDX) machine in Princeton[1] in connection with perpendicular neutral beam heating. This was a new mode of MHD type introduced and driven by the trapped fast particles. It was explained theoretically within 1 year.[2] While the fishbone mode was an internal kink mode driven by the fast particles at mode numbers of order 1, a similar instability was found for the MHD ballooning mode at much higher mode numbers.[3] The MHD ballooning was assumed to be responsible for precursors to the fishbones. In the following, we will do the same for a system of drift waves and MHD modes that contains the first high-frequency fishbone system. This means the excitation of a high-frequency fishbone as derived in Ref. 3. We also note that both slowing down and Maxwellian kinetic distribution functions were studied in Ref. 4 where it was concluded that the difference in results between these was marginal.

Recently, it has been found that resonance broadening[5–14] has an important effect on velocity space instabilities.[15–18] Resonance broadening appears as a nonlinear frequency shift which removes the linear frequency resonance to make the amplitudes finite. It was first derived by Dupree[6] by substituting equations for sidebands into the nonlinear terms, keeping only terms that are phase coherent with the central mode. As expected, the frequency broadening reduces the effect of the resonance. However, due to a fixed external fast particle source in velocity space, the kinetic resonance may survive if the external source is sufficiently strong and resonant with the waves. This process is similar to that of particle trapping, where particles are moved out of resonance. However, here, it is waves that vary their phase velocity due to nonlinear frequency shifts. As shown by Refs. 7–9, when there is no source to maintain the particle energy, resonance broadening is able to completely remove the kinetic nature of the wave-particle resonance.

Reference 7 was written for a homogeneous plasma. This was generalized in Ref. 8 where inhomogeneity was introduced. However, the result for the diffusivity was the same, the only generalization being that the linear growth rate will now be due to drift waves. The case for drift waves had already been studied by Dupree and Tetreault.[9] Our generalization is here that our derivation is non-Markovian. This leads to the same diagonal element for the diffusivity as that found by Connor and Pogutse.[19] Thus, Ref. 7 derives the same diagonal element for transport as found previously by us[20] for the energy transport and in Ref. 19 for the particle transport, both derivations using fluid theory, while Refs. 7 and 8 use fully nonlinear kinetic derivations. Thus, we can see this as a verification of our fluid closure, while Ref. 19 only verifies the nonlinear turbulence calculations using fluid theory. However, this was done by using both analytical and numerical calculations. We note the similarity between Ref. 19 and our fluid result in the saturation level of the turbulence which requires an outgoing boundary condition. This requires that strong zonal flows absorb the inward cascade in a finite system. Sufficiently strong zonal flows are obtained with reactive fluid closures as used both by us and in Ref. 19. Another fluid closure, relevant for fast particle modes, was given in Ref. 21, while our reactive closure, in combination with electron trapping, gave particle and heat pinches.[22]

In Ref. 7, we use a Fokker–Plank equation where the resonance broadening [see Eq. (7)] is due to nonlinear friction. This leads to a saturation in time of the quasi-linear behavior at $t = 1/\beta$, where β is the nonlinear friction of the mean square velocity deviation from the initial state. In the case of a resonant source, as we have for fast particle modes, it is easy to see that the quasi-linear growth of the mean square velocity deviation may continue in time corresponding to the presence of dissipative kinetic resonances.

We have here focused on aspects relevant to various physics descriptions of linear and nonlinear systems in fluid and kinetic theory. For this purpose, we have limited ourselves to the case of perpendicular neutral injection. For tangential injection, we need to consider further details of the geometry.[23–25] More geometry effects were considered in Refs. 25–27. The presence of kinetic resonances also requires us to consider effects of gyro-Landau resonances[21] for fast particle modes and the fluid closure for drift waves.[28–31]

This paper's structure is as follows. Section II describes the derivation of simultaneous equations for fast particle instabilities and drift wave turbulence. The fluid and kinetic equations are presented in Sec. III in order to make contact with previous derivations of fast particle instabilities. Section IV dedicates to the summary of the results.

II. DERIVATION OF SIMULTANEOUS EQUATIONS FOR FAST PARTICLE INSTABILITIES AND DRIFT WAVE TURBULENCE

We will here start from our earlier description of the high-frequency version of the fishbone instability.[3] This simplifies the geometrical aspects at the same time as it includes all the principal questions of combining drift wave dynamics and fast particle modes. This case assumes perpendicular neutral beam injection. When parallel injection is used, we need to consider various gap modes that depend more sensitively on the magnetic field geometry.[23,24]

An equation that very clearly shows the dependence of kinetic growth rates on nonlinear frequency shifts is that of the growth rate of the universal drift wave instability.[10]

$$\gamma = \left(\frac{\pi}{2}\right)^{1/2} \omega_{*e} \frac{\omega - \omega_{*e}}{k_\parallel v_{te}} e^{-\omega^2/(k_\parallel v_{te})^2}, \tag{1}$$

where γ is the mode growth rate, ω is the mode frequency, ω_{*e} is the electron diamagnetic drift frequency, $k_\parallel$ is the wave vector parallel to the magnetic field, and v_{te} is the electron thermal velocity. It is easy to see in Eq. (1) that for drift waves, where ω is close to ω_{*e}, the growth rate is very sensitive to a frequency shift. When the magnitude of ω_{*e} exceeds the mode frequency, the instability can be attenuated. The real eigenfrequency of drift waves is here the electron diamagnetic drift frequency shifted by Finite Larmor Radius Effects (FLR). Equation (1) was introduced merely to show how strong the effect of a frequency shift can be, and the details do not apply to fast particle modes. The same thing is true for nonlinear frequency shifts that involve quadratic nonlinearities. In a turbulent electrostatic state, we have the Fokker–Planck equation for turbulent collisions,[6–8]

$$\left(\frac{\partial}{\partial t} + v \cdot \frac{\partial}{\partial r}\right) W(X, X', t, t') = \frac{\partial}{\partial v}\left[\beta v + D^\nu \frac{\partial}{\partial v}\right] W(X, X', t, t'). \tag{2a}$$

Here the nonlinearities appears:

$$\beta = \sum_j \beta_j |\phi_j|^2, \tag{2b}$$

$$D^\nu = \sum_j d_j |\phi_j|^2, \tag{2c}$$

where W is the transition probability between states X and X' and during the time interval $t - t'$. Here, β is the nonlinear friction, while D^ν is the nonlinear diffusion coefficient in velocity space and ϕ is the electrostatic potential. In order to better understand the various terms as well as nonlinear formalism in general, we refer to Hasegawa.[10] Since the nonlinear friction is already quadratic in the fields, multiplied by the transition probability, we realize that we have a cubic nonlinearity of the same type as those stabilizing explosive instabilities. If we ignore the nonlinear friction, Eq. (2a) has the same form as an equation for quasi-linear transport.[10] Again it is only intensities, as in Eq. (2c), which play a role since other terms vanish by phase mixing. Thus, Eq. (2a) is the Fokker–Plank equation for turbulent collisions. It was obtained already by Dupree[6] and further explained in Refs. 9 and 10. We are interested in solutions of Eqs. (2a)–(2c), in particular, the transfer of energy between particles and waves. As it turns out, Eq. (2a) has analytical solutions in terms of transition probabilities.[4,7] While the diffusivity can be expressed in terms of D^ν also in quasi-linear theory,[10] the nonlinear friction is a strongly nonlinear feature.[6,7] It corresponds to a nonlinear frequency shift and, accordingly, resonance broadening.[16–18] It has a profound influence on the evolution in time. In particular, it influences the time variation of the mean square velocity deviation from the initial state.

Analytical solutions[4,6] help us to derive the variation in time of the mean square velocity deviation (velocity dispersion) as expressed in its most simple form in Ref. 13 [see Eq. (9.53)]. This was obtained for constant coefficients in the Fokker–Planck equation. However, the most important feature that velocity dispersion is constant asymptotically was verified numerically in quite general cases in Ref. 24. The analytical result is

$$\langle \Delta v^2 \rangle = \frac{D^\nu}{\beta}(1 - e^{-\beta t}). \tag{3}$$

When $\beta \ll 1$, Eq. (2a) gives the usual linear time dependence as known from quasi-linear theory. While D^ν can contain both weakly and strongly nonlinear components, β corresponds to a nonlinear frequency shift which is always strongly nonlinear. In Ref. 20, the diagonal part of our fluid diffusivity was derived. This also agreed with the result of Connor and Pogutse.[19] However, Ref. 20 also gave the full quasi-linear expression including off-diagonal parts. The main reason why we could derive a fluid result from a general, strongly nonlinear kinetic equation was the resonance broadening obtained due to the friction term.[7] The reason why quasi-linear theory works in the fluid case[20,28–30] is that the fluid velocity is several orders of magnitude smaller than the kinetic (thermal) velocity. While Eq. (3) is the result of turbulence calculations (c.f. also Ref. 32), we now recall that dissipative kinetic effects were also removed by the nonlinear frequency shift (resonance broadening) in the coherent works by Mattor and Parker[28] and Holod *et al.*[29] Thus, it appears that nonlinear frequency shifts have similar effects in coherent and turbulent systems.

Now Eq. (3) was derived for a situation without external sources except for sources of particles and energy that were required to maintain gradients in density and temperature. In the presence of external sources in velocity space (S_v), Eq. (2a) should be replaced by

$$\left(\frac{\partial}{\partial t}+v\cdot\frac{\partial}{\partial r}\right)W(X,X',t,t')=\frac{\partial}{\partial v}\left[\beta v+D^{\nu}\frac{\partial}{\partial v}\right]W(X,X',t,t')+S_v. \tag{4}$$

In the case of drift waves, a fast particle source would appear at about a factor 100 higher frequency than the drift wave frequency. Thus, it would oscillate rapidly and thus be averaged out.

$$\langle S_v\rangle = 0.$$

For modes oscillating close to the precession frequency of fast particles, however, the source in Eq. (4) will be important. Here, we need to realize that this source will give a fast particle instability. Thus, it will give growing perturbations. We can then use the same expression as for friction, which damps out instabilities but now with linear growth leading to

$$\frac{\partial W}{\partial t}=\frac{\partial}{\partial v}[s\,v]W(X,X',t,t')$$

or

$$S_v=-\frac{\partial}{\partial v}[s\,v]W(X,X',t,t'). \tag{5}$$

Here, the source contains the transition probability, which guarantees that it is resonant. It actually has the same form as the friction term but with the opposite sign. Our original Fokker–Planck equation turns out to have an analytical solution when the coefficients are constants.[5]

We then use the solution by Chandrasekhar[5] to derive the mean square velocity deviation (velocity dispersion) from the original state as shown in Ref. 7 [equation after Eq. (82)]. This was further simplified in Ref. 15 [Eq. (9.53)]. The effect of the friction (resonance broadening) can, in the case Eq. (4), without fast particles, be solved analytically as is given in Eq. (3).

We notice in Eq. (3) $\langle\Delta v^2\rangle$ (velocity dispersion) will start as proportional to t for small times. However, when friction becomes important at $t = 1/\beta$ it will flatten.

The flattening of $\langle\Delta v^2\rangle$ in Fig. 1(a) means that there is no more transfer of energy between particles and waves. Thus, a reactive fluid closure must be possible. Indeed, the further calculations in Ref. 7 showed that transport resulted in our diagonal diffusivity. Here, we included all moments with sources in the experiment, a point discussed in Ref. 13.

We can take the resonance broadening from Dupree[6] also discussed in Ref. 9,

$$\beta_{\omega}=\sqrt[3]{\frac{k_{\parallel}^2 d_{\omega-\omega'}}{3}}, \tag{6}$$

where d is the diffusion coefficient in velocity space and the frequency components are needed in a non-Markovian treatment. When we include the external source in velocity space, however, Eq. (3) is replaced by

$$\langle\Delta v^2\rangle=\frac{D^{\nu}}{\beta}\left(1-e^{-(\beta-s)t}\right). \tag{7}$$

Thus, the source is counteracting the resonance broadening. If we take the case $s=\beta$, the situation here is very similar to that of Refs. 2 and 3. The fast particles introduce a new high-frequency branch of the fluid dynamics which actually contains a kinetic resonance. For this branch, i.e., the fast particle branch, Fig. 1(a) is replaced by Fig. 1(b). We have here chosen $s=\beta$ which is a special case for the demonstration of the principle. In the quasi-linear case, dissipative kinetic resonances are active which is the case for fast particle instabilities such as Fishbones. We will here use a two-fluid description to study the combined system of drift waves and fast particles. In order to include a fast particle source in our fluid model, we need to change our fluid closure.

$$\frac{3}{2}n_i\left(\frac{\partial}{\partial t}+v_i\cdot\nabla\right)T_i+P_i\nabla\cdot v_i=-\nabla\cdot q_{*i}+i\nabla\cdot q_{\rm diss}. \tag{8}$$

Equation (8) includes our required modification in closure which can be used to derive gyro fluid models.[21] Here, $q_{\rm diss}$ is the dissipative closure term to be added to the diamagnetic heat flow, and it leads to $\gamma_{\rm diss}$ in Eq. (9). Since we know where the closure term enters in our fluid

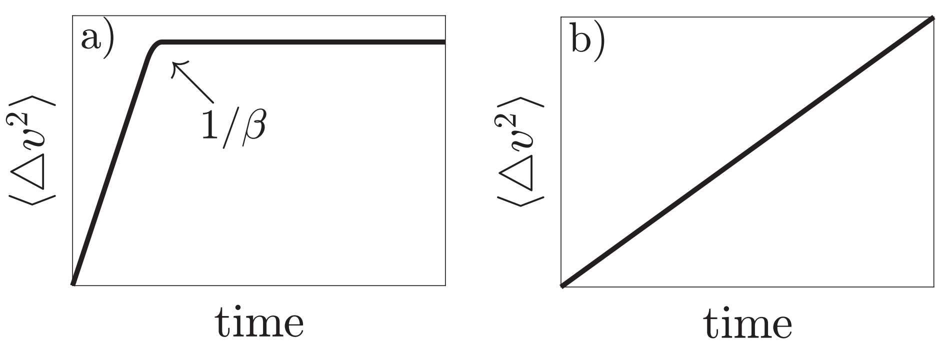

FIG. 1. (a) An illustration of the velocity dispersion over time. The original linear growth is a quasi-linear behavior, while the flattening at $t = 1/\beta$ is a strongly nonlinear effect. (b) Velocity dispersion as a function of time when external source exactly balances resonance broadening.

model, it is easy to generalize it [as obtained from Eq. (6.153) in Ref. 15]. The ion thermal diffusivity can be written as

$$\chi_{\rm i} = Re\left\{\frac{1}{\eta_{\rm i}}\left[\eta_{\rm i} - \frac{2}{3} - (1-f_{\rm t})\frac{10}{9\tau}\varepsilon_{\rm n} - \frac{2}{3}f_{\rm t}\Delta\right] \times \frac{\gamma^3/k_r^2}{(\omega_r - 5/3\omega_{\rm Di} - i\gamma_{\rm diss})^2 + \gamma^2}\right\}, \quad (9)$$

where η_i is the ratio of normalized ion temperature to normalized ion density gradients, f_t is the fraction of trapped electron, $\varepsilon_{\rm n}$ is the density gradient scale length normalized by major radius, τ is the electron-to-ion temperature ratio, k_r is the wavevector perpendicular to the magnetic field, $\omega_{\rm Di}$ is the ion magnetic drift frequency, $\gamma_{\rm diss}$ is the gyro-Landau fluid resonance, and the term Δ is due to the coupling between electrons and ions in our fluid model and is not needed here. These are taken from our derivations in Refs. 15 and 20. It is important to note here that all parts of Eqs. (8) and (9) are taken from the simplest limit of our fluid model for drift waves. For these, $\langle S_v\rangle = 0$ applies. For the fast particles, we have to use Eq. (5) which will then give the gyro-Landau resonance[21] called $q_{\rm diss}$ in Eq. (8) and accordingly $\gamma_{\rm diss}$ in Eq. (9). It will be taken from Ref. 3 but with a reduction due to resonance broadening as found above.

We also recall that our Fokker–Planck equation without an external source resulted in both the diagonal part of our diffusivity Eq. (9) if we omit the $\gamma_{\rm diss}$ and the particle diffusivity in Ref. 19. Now, we know that the external source $S_{\rm v}$ in Eq. (4) will lead to a replacement of the friction β by $\beta - s$, so we can directly make the same replacement in Eq. (9), i.e., $\gamma_{\rm diss} \sim s - \beta$, for $s > \beta$. When β "dominates," there is no dissipative term at all. This means that resonance broadening is able to completely remove wave-particle resonances.

III. FLUID RESONANCES

We now want to work with fluid and kinetic equations in order to make contact with previous derivations of fast particle instabilities. Our first observation is then that the approximation $E_\parallel = 0$ has usually been used in previous calculations. As it turns out, this is generally due to ignoring some toroidal effects. Thus, following our previous derivations for drift waves[15] but adding a gyro-Landau resonance which would be important at higher frequencies, we find starting by the density perturbation. From now on, we introduce "f" for "fast" particles.

$$\frac{\delta n_{\rm f}}{n_{\rm f}} = \left[\frac{\omega_{*\rm e} + \tau_{\rm f}\omega_{\rm pr}}{\omega - \omega_{\rm pr}} - k^2\rho_s^2 + \frac{\omega_{\rm pr}}{\omega - \omega_{\rm pr}} \times \left(\eta_{\rm f} - \frac{2}{3}\right)\frac{\omega_{*\rm e}}{\omega - 5/3\omega_{\rm pr} + i\Delta}\right]\frac{e\phi}{T_{\rm e}}, \quad (10)$$

where $\rho_{\rm s} \equiv c_{\rm s}/\omega_{\rm ci}$, is the expression for the ion Larmor radius using the electron (rather than ion) temperature ($T_{\rm e}$), $\omega_{\rm ci} = eB/m_{\rm i}$ is the ion cyclotron frequency, $c_{\rm s} \equiv \sqrt{T_{\rm e}/m_{\rm i}}$ is the speed of sound, e is the electron charge, B is the magnetic field strength, $m_{\rm i}$ is the ion mass, and $\eta_{\rm f}$ is the ratio of normalized fast ion temperature to normalized fast ion density gradients. The last term in Eq. (10) is a toroidal effect since $\omega_{\rm pr}$ represents the bounce frequency. This term is due to the fast ion temperature perturbation as seen from

$$\frac{\delta T_{\rm f}}{T_{\rm f}} = \frac{\omega}{\omega - 5\omega_{\rm Di}/3 + i\Delta}\left[\frac{2}{3}\frac{\delta n_{\rm f}}{\rm n} + \frac{\omega_{*\rm e}}{\omega}\left(\eta_{\rm f} - \frac{2}{3}\right)\frac{e\phi}{T_{\rm e}}\right]. \quad (11)$$

We have here included the modified closure in the term Δ. Thus, Eqs. (10) and (11) should be sufficient for our derivation after identifying the explicit term of the resonance. We now need to include also electromagnetic effects. We can do that by including the effect of induction in preventing electrons from reaching a Bolzmann distribution thus, we obtain from parallel electron motion:

$$\frac{\delta n_{\rm e}}{n_{\rm e}} = \frac{e\phi}{T_{\rm e}} + \frac{\omega_{*\rm e} - \omega}{k_\parallel}\frac{eA_\parallel}{T_{\rm e}}. \quad (12)$$

Combining Eq. (12) with the electron continuity equation, we get

$$\frac{eA_\parallel}{T_{\rm e}} = \frac{k_\parallel(\omega - \omega_{*e})}{\omega(\omega - \omega_{*e}) - k^2\rho_s^2 k_\parallel^2 v_A^2}\frac{e\phi}{T_e}. \quad (13)$$

By substituting the value of $A_\parallel$ into Eq. (12), it becomes

$$\frac{\delta n_{\rm e}}{n_{\rm e}} = \frac{e\phi}{T_{\rm e}}\left[1 + \frac{(\omega_{*\rm e} - \omega)^2}{\omega(\omega_{*\rm e} - \omega) - k^2\rho_{\rm s}^2 k_\parallel^2 v_{\rm A}^2}\right], \quad (14)$$

where $v_{\rm A}$ is the Alfvén velocity. The main ion density equation is expressed as

$$\frac{\delta n_{\rm i}}{n_{\rm i}} = \frac{e\phi}{T_{\rm e}}\left[\frac{\omega_{*\rm e}(1 - k^2\rho_{\rm i}^2)}{\omega - \omega_{\rm Di}} - k^2\rho_{\rm s}^2\right], \quad (15)$$

where $\rho_{\rm i}$ is the ion Larmor radius. Now using quasineutrality between electrons, main ions, and fast ions, we obtain

$$\left[1 + \frac{(\omega_{*\rm e} - \omega)^2}{\omega(\omega_{*\rm e} - \omega) - k^2\rho_{\rm s}^2 k_\parallel^2 v_{\rm A}^2}\right] = \left[\frac{\omega_{*\rm e} + \tau_{\rm f}\omega_{\rm Di}}{\omega - \omega_{\rm Di}} - k^2\rho_s^2 + \frac{\omega_{\rm Di}}{\omega - \omega_{\rm Di}}\left(\eta_{\rm i} - \frac{2}{3}\right)\frac{\omega_{*\rm e}}{\omega - \frac{5}{3}\omega_{\rm Di}}\right] + \left[\frac{\omega_{*\rm e} + \tau_{\rm f}\omega_{\rm pr}}{\omega - \omega_{\rm pr}} - k^2\rho_s^2 + \frac{\omega_{\rm pr}}{\omega - \omega_{\rm pr}}\frac{\left(\eta_{\rm f} - \frac{2}{3}\right)\omega_{*\rm e}}{\omega - \frac{5}{3}\omega_{\rm pr} + i\Delta}\right]. \quad (16)$$

We note that the dissipative resonance exists only for the fast ions since they are resonant with the fast ion source. The dissipative resonance disappears from the other terms because of Eq. (5) the dissipative term is proportional to $s - \beta$.

We first ignore the fast particle part of Eq. (16) and then arrive at

$$\frac{\omega_{\rm Di}\omega_{*\rm e}}{k^2\rho_{\rm s}^2}\left(\frac{\omega_{*\rm e}}{\omega} - 1\right) + k_\parallel^2 v_{\rm A}^2\left(1 - \frac{\omega_{\rm Di}}{\omega}\right) - (\omega - \omega_{*\rm e})(\omega - \omega_{\rm Di}) = \frac{2}{\tau}\omega_{*\rm e}(\omega_{*\rm e} - \omega) - k_\parallel^2 v_{\rm A}^2 \times \left[\frac{\omega_{*\rm e}}{\omega} + k^2\rho_{\rm s}^2\left(1 - \frac{\omega_{\rm Di}}{\omega}\right)\right]. \quad (17)$$

We can here check the MHD limit by taking the limit $\omega \gg \omega_*, \omega_{\rm Di}$. We then obtain standard MHD ballooning modes

$$\omega^2 = k_\parallel^2 v_{\rm A}^2 - D, \quad (18a)$$

$$D = \frac{\omega_{*e}\omega_{De}}{k^2\rho_s^2}. \quad (18b)$$

Now looking at the fast particle part, we obtain

$$\left[\omega(\omega_{*e}-\omega) - k^2\rho_s^2 k_\parallel^2 v_A^2 + (\omega_{*e}-\omega)^2\right]\frac{e\phi}{T_e} = f_f\frac{e\phi}{T_e}\left[\frac{\omega_{*ef}+\tau_f\omega_{pr}}{\omega-\omega_{pr}} - k^2\rho_s^2\right] + f_f\frac{\omega_{pr}}{\omega-\omega_{pr}}\frac{\delta T_f}{T_f}, \quad (19)$$

where $f_f = n_f/n_e$ is the fraction of fast ions. Including the temperature perturbation from Eq. (11), we get

$$\omega(\omega-\omega_{*e}) - k_\parallel^2 v_A^2\left(1 - \frac{\omega_{*ef}}{\omega-\omega_{pr}} + k^2\rho_s^2\right) = -(\omega_{*e}-\omega)\left[\frac{\omega_{*e}}{k^2\rho_f^2} - \frac{D_f}{\omega-\omega_{pr}+i\Delta}\right], \quad (20a)$$

$$D_f = \frac{\omega_{*f}\omega_{Df}}{k_\perp^2\rho_f^2}, \quad (20b)$$

contains the fast particles fraction f_f and here Δ includes also the resonance broadening. We are now in a position to write down our final stability condition,[3]

$$\frac{i[\omega(\omega-\omega_{*i})]^{1/2}}{\omega_A} = \delta W_{fluid} + \delta W_{fast}. \quad (21)$$

We here introduced a normalization with the Alfvén frequency ω_A which means that the different parts here are dimensionless. We have already seen how the fluid part gives us the two fluid parts of the MHD energy change. In our full description, the parallel electric field is included. Also, the fast particle part includes the parallel electric field and resonance broadening. Thus, Eq. (21) will turn into the equation for high-frequency fishbones in Ref. 3. We note that Eq. (21) contains both the fluid, drift wave, and the fast particle part δW_{fast}. These have been treated differently. Thus, while δW_{fluid} uses isothermal electrons, δW_{fast} uses the full kinetic integral for the fast particles as taken from Ref. 3.

If we take the limit when ω is much larger than the drift frequencies. We then have to evaluate the fast particle resonance, which will be the same as in Ref. 3, except for the fact that we have here included resonance broadening. Thus, we can now use the resonant part of δW_{fast} as given in Ref. 3 [see Eq. (29) in Ref. 3].

$$\delta W_{fast} = \frac{\pi q^2}{2s}\hat{\beta}_f I_0 \frac{\omega}{\omega_{pr}}\ln\left(1-\frac{\omega_{pr}}{\omega}\right). \quad (22a)$$

Here we introduce:

$$\hat{\beta}_f = \zeta, \quad (22b)$$
$$\zeta = (s-\beta)/s. \quad (22c)$$

Thus, we have modified δW_{fast} from Ref. 3 to include a reduction due to resonance broadening. Here, ζ represents the effect of resonance broadening, β_f is the fast ion pressure to magnetic pressure ratio, and I_0 represents geometry effects partly due to solving the linear eigenvalue problem for MHD ballooning modes. However, also δW_{fluid}, which is a generalization of δW_{MHD} in Ref. 3 to include two fluid effects, has been improved partly due to solving the linear eigenvalue problem for MHD ballooning modes as seen in Refs. 26 and 27.

In our calculation for the high-frequency fishbone, we found[3]

$$I_0 = \frac{0.5}{K_{b0}}\left(\alpha_0 I'(\alpha_0) + I(\alpha_0)\frac{\omega_{*f}}{\bar{\omega}_{df}}\right), \quad (23)$$

where

$$K_{b0} = \left(\frac{2R}{r}\right)^{0.5} K(k_0^2)/\pi,$$
$$I(\alpha_0) = \left(\frac{2R}{r}\right)^{0.5}\left[2E(k_0^2) - K(k_0^2)\right]^2/\pi K(k_0^2),$$
$$k_0^2 = \left(1+\frac{r}{R}-\alpha_0 B_0\right)\frac{r}{2R},$$
$$\bar{\omega}_{Df} = -nq\left[2E(k_0^2)/K(k_0^2) - 1\right]/R\,r\,\Omega_c.$$

Continuing to follow the ordering in Ref. 3, we have

$$\omega_{pr} = \bar{\omega}_{Df} \gg \omega_{*e}; \quad \bar{\omega}_{Df} < \omega_{Df},$$

where r is the minor radius, R is the major radius, $\alpha_0 = \omega/\omega_{pr}$, Ω_c is the cyclotron frequency, and $E(k_0^2)$ and $K(k_0^2)$ are complete elliptic integrals. Here, ω_{pr} is the precession frequency of the fast particles which equal the bounce-averaged fast particle magnetic drift frequency, $\bar{\omega}_{Df}$ and ω_{*e} are the diamagnetic drift frequency of the core plasma which has much lower temperature. Furthermore, the bounce averaged magnetic drift is smaller than the magnetic drift itself since bounce averaging reduces its magnitude. This ordering is consistent with the PDX experimental data from Ref. 3. The imaginary part of Eq. (21) directly gives us the term Δ but here proportional to $s-\beta$.

However, we can also go in the other direction, including more physics. In our full fluid model,[15,31] we use

$$\frac{eA_\parallel}{T_e} = \frac{k_\parallel(\omega-\omega_{*e})}{\omega(\omega-\omega_{*e}) + \omega_{De}(\omega_{*eT}-\omega) - \frac{mk_\parallel T_e}{e^2Brn_0}\frac{\partial J_{\parallel 0}}{\partial r} - k^2\rho_s^2 k_\parallel^2 v_A^2\left(1-\frac{i(\omega-\omega_{De})}{k_\parallel^2 D_e}\right)}\frac{e\phi}{T_e}. \quad (24)$$

Using Eq. (24) in Eq. (12) means that we include also kinetic ballooning modes and peeling modes in our description. Thus, while fast particle-driven modes appear close to the boundary of MHD ballooning modes in Ref. 3, they will here appear close to the combined boundary of kinetic ballooning modes and peeling modes.

IV. SUMMARY

We have here included the effects of fast particles in our fluid model for drift waves. In doing this, we have entered a source term into the Fokker–Planck equation previously used for the derivation of our fluid model. This source has to have a frequency comparable to the eigenmodes we study. Thus, it enters only because we are here considering eigenmodes with frequencies of the order of the precession frequency of the energetic trapped particles, which is typically about two orders of magnitude larger than that of drift waves. We have here included two fluid effects containing the parallel electric field and also kept the resonance broadening, which reduces the effect of the fast particle drive. The way we include an extra source connects closely to the discussion in Ref. 13. There is a rather detailed discussion of how external sources influence our system. This discussion can easily be extended to include an external fast particle source. This was the main new content of our work, from which it follows that our standard fluid closure is valid when resonance broadening dominates over the external source, while we need to include a dissipative kinetic source (like a Landau fluid resonance) when the external kinetic source dominates over resonance broadening.

Our work applies in its respective limits to both drift wave theory and the theory of fast particle instabilities. For fast particles, we refer to Refs. 4 and 25 for a comparison between results for Maxwellian and slowing-down distributions. Reference 25 also includes the effects of geometry, which are mainly discussed in Refs. 23 and 24 for gap modes. The new results for fast particle modes are mainly resonance broadening and two fluid effects like parallel ion motion.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Jan Weiland: Writing – original draft (equal); Writing – review & editing (equal). **Tariq Rafiq:** Funding acquisition (lead); Writing – original draft (equal); Writing – review & editing (equal). **Eugenio Schuster:** Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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