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Communication

Unveiling the Significance of Correlations in K-Space and Configuration Space for Drift Wave Turbulence in Tokamaks

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Abstract: Turbulence and transport phenomena play a crucial role in the confinement and stability of tokamak plasmas. Turbulent fluctuations in certain physical quantities, such as density or temperature fluctuations, can have a wide range of spatial scales, and understanding their correlation length is important for predicting and controlling the behavior of the plasma. The correlation length in the radial direction is identified as the critical length in real space. The dynamics in real space are of significant interest because transport in configuration space is primarily focused on them. When investigating transport caused by the $\mathbf{E} \times \mathbf{B}$ drift, the correlation length in real space represents the size of $\mathbf{E} \times \mathbf{B}$ whirls. It was numerically discovered that in drift wave turbulence, this length is inversely proportional to the normalized mode number of the fastest growing mode relative to the drift frequency. Considerable time was required before a proper analytical derivation of this condition was accomplished. Therefore, a connection has been established between phenomena occurring in real space and those occurring in k-space. Although accompanied by a turbulent spectrum in k-space with a substantial width, transport in real space is uniquely determined by the correlation length, allowing for accurate transport calculations through the dynamics of a single mode. Naturally, the dynamics are subject to nonlinear effects, with resonance broadening in frequency being the most significant nonlinear effect. Thus, mode number space is once again involved. Resonance broadening leads to the detuning of waves from particles, permitting a fluid treatment. It should be emphasized that the consideration here involves the total electric field, including the induction part, which becomes particularly important at higher beta plasmas.

Keywords: turbulence and transport modeling; magnetic confinement; correlation length; resonance broadening; tokamaks; drift waves

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1. Introduction

The description of turbulence in tokamak plasmas is usually made in k-space [1–15]. However, the complementary description in real space is sometimes useful [1,2]. This is particularly the case when we want to describe drift wave transport. We are particularly interested in high-beta cases where electromagnetic effects cannot be ignored. This means that we need to consider the asymptotic behavior of the eigenfunction in this limit. As it turns out, the asymptotic behavior of the electrostatic case can still be used [3,14]. Concerning high-beta, we note that linearly our present model contains the second stability regime of the ideal magnetohydrodynamic (MHD) ballooning mode [12,13]. We also note the ability of our model to describe the L-H transition [15] where the system spontaneously and without adjustments enters an H-mode where the gradient of the pedestal is in the second stability regime of MHD ballooning modes. The fact that our quasilinear description is well at the level of nonlinear simulations [16] is confirmed by the agreement of the H-mode simulations with [15,17].

2. Basic Equations

We begin with the expression for electron density perturbation Equation (1), which is derived from the collisionless electromagnetic parallel electron equation of motion, the ion energy equation, electron density, and Ampere's law in Ref. [4]. The expression depends on the vector potential component $A_{\|}$, so the deviation from a Boltzmann expression is due to the electromagnetic induction. We derived an expression for the correlation length in the electrostatic case in Ref. [3]. It focused on how the correlation length in real space describes the width of the eigenfunction.

$$\frac{\delta n_e}{n_e} = [1 + \delta(\omega)]\frac{e\varphi}{T_e}, \tag{1a}$$

where:

$$\delta(\omega) = \frac{(\omega_{*en} - \omega)^2}{\omega(\omega_{*en} - \omega) + \omega_{De}(\omega - \omega_{*ep}) + k^2\rho^2 k_{\|}^2 {v_A}^2}. \tag{1b}$$

Here, δn_e is the electron density (n_e) perturbation, φ is the electrostatic potential, e is the electronic charge, T_e is the electron temperature, k is the wave propagation vector, $k_{\|}$ is wave vector parallel to the magnetic field, ω is the the mode frequency, ω_{*en} is the electron diamagnetic drift frequency, ω_{*ep} is the diamagnetic drift and is due to the inhomogeneity of electron pressure, ρ is the ion Larmor radius, v_A is the Alfvén velocity, and ω_{De} is the magnetic drift frequency. The expression for the magnetic drift is standard. It has different signs for ions and electrons and is due to the inhomogeneity of the background magnetic field.

The above-mentioned quantities are standard and defined in Ref. [4]. To generalize this result, we employ a more general parallel vector potential, $A_{\|}$, that includes the missing terms such as current density gradient (peeling term) and collisionality. A more general vector potential is taken from Ref. [7]. As seen in Equation (2), this expression does not contain an electromagnetic correction to the asymptotic δ, and thus we basically have the same mode profile as in the electrostatic case. This also follows from the investigations in Refs. [4–6]. We also note that the effect of magnetic compression ($\delta B_{\|}$) can be included as a modification of the magnetic drift frequency, as shown in Ref. [7] Equation (6.21). Further inclusion of magnetic compression would involve the compressional Alfvén wave. This is motivated only in connection with heating. The gauge condition is the standard $divA = 0$.

$$\frac{eA_{\|}}{T_e} = \frac{k_{\|}(\omega - \omega_{*e})}{\omega(\omega - \omega_{*e}) + \omega_{De}(\omega_{*ep} - \omega) - \frac{mk_{\|}T_e}{e^2Brn_0}\frac{\partial J_{\|}}{\partial r} - k_{\perp}^2\rho_s^2 k_{\|}^2 v_A^2\left(1 - \frac{i(\omega - \omega_{De})}{k_{\|}^2 D_e}\right)}\frac{e\varphi}{T_e}. \tag{2}$$

Here, $D_e = T_e/(m_e \nu_e)$, m_e is the electron mass, ν_e is the electron-ion collision frequency, m is the poloidal mode number, r is the minor radius, and $J_{\|}$ is the background current. Of particular interest here is the direction of the particle flow. A rather general formula was derived in Ref. [4]. It is:

$$D = (1 - f_t)D_1 {T_e}^{1.5}\varepsilon_n {\varpi_i}^2 \frac{Im\delta(\omega)}{k_r\rho_s}, \tag{3}$$

where we introduced

$$\varpi = \frac{\omega}{\omega_{De}} = \varpi_r + i\varpi_i, \tag{4}$$

$\epsilon_n = 2L_n/R$, L_n is the electron density gradient scale length, R is the major radius, $\varpi_r(\varpi_i)$ is the linear mode real frequency (growth rate), f_t is the fraction of trapped electrons, and k_r is the radial propagation wave vector. Here, D_1 is our standard normalization of diffusivities in our code depending, e.g., on the major radius.

2.1. Effects of Current Gradient

We are now looking for imaginary parts of δ that will give a systematic particle flow. We begin by ignoring collisions. Then, we have:

$$Im\delta(\omega) = \frac{\varpi_i}{|N|^2}\left\{\varpi_i{}^2\left(\frac{1}{\varepsilon_n}-1\right)+\left(\varpi_r-\frac{1}{\varepsilon_n}\right)\left[\left(\frac{1}{\varepsilon_n}-1\right)\left(\varpi_r-\frac{1}{\varepsilon_n}\right)-2\Omega_A-\frac{mk_\parallel T_e}{e^2Brn_0}\frac{\partial J_\parallel}{\partial r}+2\frac{\eta_e}{\varepsilon_n}\right]\right\}, \tag{5a}$$

$$\Omega_A = k^2\rho^2k_\parallel^2v_A^2/\omega_{De}^2, \tag{5b}$$

$$N = \varpi_r\left(1+\frac{1}{\varepsilon_n}\right)-\varpi_r{}^2+\varpi_i{}^2-\frac{1+\eta_e}{\varepsilon_n}+\Omega_A+\frac{mk_\parallel T_e}{e^2Brn_0}\frac{\partial J_\parallel}{\partial r}+i\varpi_i\left(1+\frac{1}{\varepsilon_n}-2\varpi_r\right), \tag{5c}$$

where η_e is the ratio of electron temperature to electron density gradient. From Equation (5a), we notice that we have a more likely particle pinch if the imaginary part of the profile is flat, which is generally the case in the core region of the plasmas. However, here too, the current gradient is relevant. It may have either sign, so we have to look at a particular experimental case in order to draw conclusions. From Equation (2), we notice that collisions may also enter here. However, they enter as an imaginary part and could add to the imaginary part of ω.

2.2. Effects of Collisions

Another possibility for obtaining a steady flow is through collisions. If we treat the frequency as real, the imaginary part of δ can be written as:

$$Im\delta(\omega) = \frac{AC}{B^2+C^2}, \tag{6}$$

where

$$A = \frac{\omega_{*en}-\omega}{\omega}\left[\omega(\omega_{*en}-\omega)+\omega_{De}(\omega-\omega_{ep})+k^2\rho^2k_\parallel^2v_A^2\right],$$

$$B = \omega\ (\omega_{*en}-\omega)+\omega_{De}(\omega-\omega_{ep})+k^2\rho^2k_\parallel^2v_A^2,$$

$$C = k^2\rho^2k_\parallel^2v_A^2\frac{\omega-\omega_{De}}{k_\parallel^2D_e}.$$

Also here, an effect of the sign of frequency can be seen. If magnetic drift does not dominate, we expect that the trapped electron mode (TEM) will give way to an outward flow, whereas if magnetic drift dominates, TEM will give way to particle pinch. We now note that our model has been able to describe particle pinches in Tore Supra [9], and simulations closely resembling these were also performed with QualiKiz [8]. As it turned out, these results were also consistent with our first model giving a particle pinch [10] and are also supported by similar phenomena in the levitated dipole experiment and particle motion in the ionosphere [11].

3. Correlation Length

We are particularly interested in the correlation length. This is actually the effective size of eddies. As found originally in our first paper on transport, the effective mode number that gives transport is the mode number at the maximum growth rate as normalized by the drift frequency. The saturation condition is:

$$\omega_i\delta T = \mathbf{v_E}\cdot\nabla\delta T, \tag{7}$$

where δ indicates perturbation. We now recall that smaller eddies tear apart larger eddies since their $\mathbf{E}\times\mathbf{B}$ flow is stronger due to the larger mode number. However, for larger mode numbers, the electric field strength is reduced due to the smaller growth rate. Thus, our condition becomes:

$$\varpi_i = \frac{\omega_i}{\omega_{De}} = \frac{\mathbf{V_E} \cdot \mathbf{k}}{\omega_{De}}. \tag{8}$$

In Equation (8), we see that the only mode number dependence here is the $\mathbf{E} \times \mathbf{B}$ drift. Thus, the maximum of the growth rate as normalized by the drift frequency is at the maximum of the $\mathbf{E} \times \mathbf{B}$ drift. To determine the actual value of the correlation length, we start from the eigenfunction, which has the form:

$$\varphi \propto e^{-\alpha\theta^2}, \tag{9a}$$

where:

$$\alpha = -i\varpi_i k^2 \rho^2 \hat{s} q, \tag{9b}$$

θ is the extended angle, q is the safety factor, $\hat{s} = \sqrt{2s - 1 + \kappa^2(s-1)^2}$ in which s is the magnetic shear, and κ is the elongation. In finding the scaling of the correlation length, we were guided by the parametric dependence of the eigenfunction in Equation (9).

Simulations gave us the scaling [3,18].

$$fls = \left(0.7 + \frac{2.4}{7.14 q\hat{s} + 0.1}\right) fl, \tag{10a}$$

$$k_\theta \rho_s = \sqrt{\frac{2fls}{1 + 1/\tau}}, \tag{10b}$$

where $fl = 0.1$, which is the finite Larmor radius (FLR) parameter usually used in slab calculations. Here, k_θ is the inverse correlation length and τ is the temperature ratio. This scaling was tested extensively with good results in Ref. [3]. Another point to be considered here is the fact that we used a fluid model. Clearly, the eigenfunction could have been modified by kinetic wave-particle resonances. However, our model is a fluid model where kinetic resonances do not appear. This is motivated by going into the nonlinear regime where resonance broadening [19] is active. The resonance broadening was first applied to our system in Ref. [20] and has recently been applied to a generalized system including fast particles in Ref. [21].

We also recall that using Equation (10) as a single wave number in simulations with many modes was verified numerically in Ref. [1]. In reality, a few modes around the correlation length contribute to the transport, but we here see that we can replace these by using only the correlation length, which means that we are looking at transport in real space as given by the $\mathbf{E} \times \mathbf{B}$ rotation at its maximum. There are also other cases where our model gives us flows in real space, such as at internal transport barriers [22] and at the H-mode barrier [15,23].

4. High Confinement Mode (H-Mode)

H-mode is characterized by improved plasma stability, increased energy and particle confinement, and reduced transport of heat and particles across the magnetic field. H-mode enables longer plasma pulses and higher fusion power output, bringing us closer to the goal of sustained, controlled nuclear fusion as a clean and abundant energy source.

The H-mode is achieved through a combination of several factors, including the formation of an edge transport barrier (ETB), the presence of a steep pressure gradient, and the establishment of a sheared plasma flow near the plasma edge. As we have stressed several times, zonal flows are much stronger in reactive systems than when dissipation is included [7,24]. This helps one to obtain both internal transport barriers (ITB), the ETB, and the Dimits nonlinear upshift [25]. The result is visible in the expression for the ion thermal conductivity.

$$\chi_i = Re\left[\frac{1}{\eta_i}\left(\eta_i - \frac{2}{3} - (1-f_t)\frac{10}{9\tau}\varepsilon_n - \frac{2}{3}f_t\Delta_i\right)\frac{(\omega_i - \omega_{E\times B})^3/k_r^2}{(\omega_r - \frac{5}{3}\omega_{Di} - i\gamma_{diss})^2 + (\omega_i - \omega_{E\times B})^2}\right], \tag{11}$$

where Δ_i is the contribution from trapped electrons to the ion temperature pinch [7]:

$$\Delta_i = \frac{1}{N} \quad \Big\{|\hat{\omega}|^2\Big[|\hat{\omega}|^2(\varepsilon_n - 1) + \hat{\omega}_r\varepsilon_n\left(\frac{14}{3} - 2\eta_i - \frac{10}{3}\varepsilon_n\right) + \frac{5}{3}\varepsilon_n^2\left(-\frac{11}{3} + 2\eta_e + \frac{7}{3}\varepsilon_n\right) - \frac{5}{3\tau}\varepsilon_n^2\left(1 + \eta_e - \frac{5}{3}\varepsilon_n\right)\Big] + \frac{50}{9\tau}\hat{\omega}_r\varepsilon_n^3(1-\varepsilon_n) - \frac{25}{9\tau}\varepsilon_n^4\left(\frac{7}{3} - \eta_e - \frac{5}{3}\varepsilon_n\right)\Big\},$$

$$N = \left(\hat{\omega}_r^2 - \hat{\omega}_i^2 - \frac{10}{3}\hat{\omega}_r\varepsilon_n + \frac{5}{3}\varepsilon_n^2\right)^2 + 4\hat{\omega}_i^2\left(\hat{\omega}_r - \frac{5}{3}\varepsilon_n\right)^2,$$

and γ_{diss} is the Landau damping or magnetic drift resonance [26].

$$\gamma_{diss} = -\frac{3}{4}\left(\sigma_t\sqrt{2} - i\right)\omega_{\mathrm{Di}} - \frac{2}{\sqrt{\pi}}k_\parallel\sqrt{\frac{2T_i}{m_i}}\sigma_s, \quad \sigma_t = \frac{\omega_{\mathrm{Di}}}{|\omega_{\mathrm{Di}}|}, \quad \sigma_s = \frac{k_\parallel}{|k_\parallel|}.$$

Here, η_i is the ratio of the ion temperature and density gradients, and $\omega_{E\times B}$ is the flowshear due to background flow and the hat quantities are normalized with the electron diamagnetic drift frequency. Since marginal stability typically occurs for ω_r close to $5/3\omega_{Di}$, we realize that χ_i will be very sensitive to the fluid closure.

The subtraction of $\omega_{E\times B}$ from the linear growth rate in Equation (11) means that we use Waltz's rule [27] for stabilization due to flow shear. Also, the flow shear is sensitive to the fluid closure according to the rotation:

$$\Gamma_p = \langle V_{Er}V_\theta\rangle = -\frac{1}{2}D_B^2 k_r k_\theta \hat{\phi}^*\left[\hat{\phi} + \frac{1}{\tau}\hat{P}_i\right] + c.c, \tag{12}$$

where Γ_p off-diagnoal poloidal momentum flux; V_{Er} is the radial component of $\mathbf{E}\times\mathbf{B}$ drift; V_θ is the poloidal flow velocity; $D_B = \rho_s c_s$, c_s is the sound speed; k_θ is the polidal wavenumber; $\hat{\phi} = e\phi/T_e$; and $\hat{P}_i = \delta P_i/P_i$. In Equation (12), we note the dependence of ion temperature (T_i) on ion pressure (P_i), which is also very sensitive to the fluid closure, through the formula:

$$\hat{T}_i = \frac{\omega}{\omega - \frac{5}{3}\omega_{Di} + i\gamma_{diss}}\left[\frac{2}{3}\hat{n}_i + \frac{\omega_{*e}}{\omega}\left(\eta_i - \frac{2}{3}\right)\hat{\phi}\right]. \tag{13}$$

Here, $\hat{T}_i = \delta T_i/T_i$ and $\hat{n}_i = \delta n_i/n_i$. From Equation (13), we conclude that the temperature perturbation and, accordingly, the flow are much stronger in a reactive fluid model. This is the reason why we have been able to recover both the ITB (poloidal spinup), the ETB (H-mode), and the Dimits shift [25] with our reactive fluid model. Here, we point out that both kinetic ballooning modes and peeling modes are active in the H-mode barrier. We thus realize that our new knowledge about correlations in electromagnetic systems, mentioned earlier in this work, is essential. We also stress that we have good agreement with strongly nonlinear turbulence simulations for the MHD ballooning parameter (α) in transport barriers seen in Alcator C-mod [17].

5. Zonal Flows

As mentioned above, zonal flows are often important. They generally arise close to fluid resonances. Our rule for including the appearance of zonal flows is to use the Waltz rule [27]. This means that we subtract the $\mathbf{E}\times\mathbf{B}$ shearing rate from the growth rate. This procedure gives us the L-H transition as well as ITB and the Dimits nonlinear upshift. The

Waltz rule gives us an excitation of zonal flows, the strength and location of which depend strongly on the fluid closure in fluid models.

6. Discussion

This paper explores the impact of electromagnetic effects on various plasma phenomena, including the correlation length, particle pinches, and the L-H transition. A noteworthy finding is that electromagnetic effects do not alter the asymptotic behavior, specifically for large FLR effect, of the eigenfunction. This behavior determines the shape of the eigenfunction, thereby influencing the correlation length. The advantage of fluid models over fully kinetic models is the vast difference in computing time. Here, it is not only the fact that fluid models do not need to work in six dimensional phase space but that they only work in three-dimensional configuration space. Another advantage of using fluid models is that it is sufficient to use a quasilinear theory, while kinetic models require strongly nonlinear effects such as resonance broadening. This is because kinetic velocities are several orders of magnitude larger than fluid velocities. In fact, it is resonance broadening that reduces kinetic theory to fluid theory, where we can truncate at the irreducible fourth moment [28]. We can also compare it with an electromagnetic kinetic paper [29].

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