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Remediation of time-delay effects in tokamak axisymmetric control loops by optimal tuning and robust predictor augmentation

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ABSTRACT

It is sometimes incorrectly assumed that, because superconducting tokamaks already have significant intrinsic or imposed sources of control delay, introducing extra delays/lags into the axisymmetric control loops will have negligible detrimental impact on the plasma control. This study exposes and quantifies the detrimental effects imposed by time delays/lags in the control loop in superconducting tokamaks, using as an example the plasma current control and radial position control in a vertically stable circular plasma in the KSTAR tokamak. Delays and lags in the power supplies, data acquisition, and vessel structure are taken into account. Optimal tuning of PID controllers in combination with an ohmic-flux control strategy is proposed as a possible method for remediating the negative effects of time delays/lags. In addition, an augmentation of the control loop by the introduction of a robust predictor has been proposed to improve the performance of the time-delayed closed-loop system when the amount of delay/lag in the loop is unknown. The Nyquist dual locus technique based on the Argument Principle in complex theory is employed to assess stability of the optimally tuned closed-loop system in the presence of time delays.

1. Introduction

With the introduction of fully superconducting tokamaks comes the need to understand how to operate and control plasmas within these devices, given new constraints imposed by superconducting PF coils: (i) there is a concern about AC losses triggering coil quench; (ii) the minimum distance of coils from the plasma is increased due to cryogenic insulation requirements; (iii) there is a greater emphasis on minimizing the number of control coils due to cost; (iv) passive structures are often more conductive, due to requirements for increased structural strength, multiple conducting walls, or intentional placement of highly conductive passive conductors near the plasma to reduce the growth rate of instabilities. All of these changes from present devices tend to change the plasma shape control properties, several of them negatively because of increased delays in responding to plasma disturbances. Since it is not obvious what constitutes an acceptable amount of delay, we have begun a study of this issue in an attempt to provide guidance to designers of external systems (power supplies, control computers, and communication networks) regarding acceptable pure delays (and also phase lags) contributed by these systems. This study has been carried out using models of the KSTAR (Korea superconducting tokamak advanced research) tokamak [1].

In this work we consider a vertically stable circular plasma. Two PID controllers are synthesized based on a decomposition of control action into ohmic flux and vertical field to control plasma current and radial position, respectively. Extremum seeking is proposed for optimal tuning of the PID gains in presence of time delays. Extremum seeking, which is a nonmodel-based method, iteratively modifies the arguments of a cost function (in this application, the PID parameters) so that the tracking error is minimized [2] (see references therein for alternative PID tuning methods). In addition, an augmentation of the control loop by the introduction of a predictor has been proposed to improve the performance of the time-delayed closed-loop system. It is shown that the proposed predictor is robust against uncertainties in the values of the delays. The closed-loop stability analysis is carried out using the dual-locus diagram (also called Satche diagram) method [3]. The dual-locus diagram method is an extension or a variant of the well-known Nyquist diagram, and is also based on the celebrated Argument Principle in complex theory. The dual-locus diagram method is simple, intuitive and quite effective in assessing stability of time-delay systems when the time delays appear in only one of the loci.

2. Plasma response model and control approach

The system composed of plasma, shaping coils, and passive structure can be described using circuit equations derived from Faraday's Law, and radial and vertical force balance relations for a particular plasma equilibrium. In addition, rigid radial and vertical

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displacement of the equilibrium current distribution is assumed, and a resistive plasma circuit equation is specified [4]. The result is a circuit equation describing the linearized response, around a particular plasma equilibrium, of the conductor–plasma system to voltages on active conductors.

Due to the vertical stability of the circular plasma the two primary parameters of interest from a control point of view are the radial position and plasma current. Each of these parameters is controlled by its own PID controller. G_p^r and G_d^r stand for the proportional and derivative gains, respectively, of the radial position PD controller (no integral action). G_p^i and G_i^i stand for the proportional and integral gains, respectively, of the plasma current PI controller (no derivative action). The radial position of the plasma is controlled using poloidal field coil PF7 (PF7U and PF7L connected in series). The plasma current is controlled using the ohmic current vector I_{ohm} [5]. The I_{ohm} vector of poloidal field currents ideally produces zero field, or equivalently constant flux, across the plasma. This constant flux, usually referred to as ohmic flux, drives the plasma current without affecting the shape (radial position in our case) of the original equilibrium. The concept of ohmic flux, which is common one in tokamak plasma physics, ideally decouples the plasma current and radial position control loops but cannot be produced by a finite set of control coils. Practically, some coupling always remains, but this coupling is treated as a disturbance by the "decoupled" control loops.

Extremum seeking, a real-time non-model-based optimization technique, is applicable in situations where there is a nonlinearity in the control problem, and the nonlinearity has a local minimum or a maximum. The parameter space can be multidimensional. Here, we use extremum seeking for iterative optimization of the PID gains (see [2]) G_p^r, G_d^r, G_p^i and G_i^i of the radial position PD controller and the plasma current PI controller to minimize the tracking error, i.e., $J = \left(\int_{t_i}^{t_f} ((R^{ref} - R)/K_R)^2 + ((I_p^{ref} - I_p)/K_I)^2 dt\right)^{1/2}$. The weights K_R and K_I were defined so that 1 cm of error between the radial position R and its reference value R^{ref} gives the same tracking error value as 2.5 kA of error between the plasma current I_p and its reference I_p^{ref}. Fig. 1(a) shows the plasma discharge controlled by optimal controller gains ($G_p^r = 144960$, $G_d^r = 0.7384$, $G_p^i = 0.0255$, $G_i^i = 0.0007195$) that minimize J for the no-delay case ($\tau_o = 0$).

3. Time delay/lag effects

The first study introduces a pure delay $e^{-\tau_o s}$ into the plasma control system (PCS), where s denotes the Laplace variable and τ_o the time delay. We study the effect of the time delay when the optimal gains obtained for the ideal no-delay case are implemented for the controllers. By introducing a delay of 1 ms into the PCS we find that the responses of the radial position and plasma current do not vary much from the no-time-delay case. However, at 3 ms, the time delay starts showing its effect on the system. With a delay of 5 ms, both responses have deteriorated (Fig. 1(b)). The radial position response and the plasma current response are both exhibiting a great deal of oscillation. However, both parameters are still roughly tracking the reference values on average. With a time delay of 7 ms, control is essentially lost. The second study introduces a lag $1/(\tau_o s + 1)$ into the power supplies. By simulating the system with non-ideal power supplies we obtain similar results to those obtained with pure delays in the PCS. The third case study involves artificially modifying the vessel element resistances. Smaller vessel resistances will have the effect of a greater delay or lag on the system. When the vessel resistance is at 90% of the design values, the plasma current response tracks the reference value well. At 50% of the design values, the plasma current response has deteriorated significantly and a large delay can be seen having an effect on the plasma current response. By the time the vessel resistance reaches 30% of the design values, essentially all plasma current control is lost.

4. PID optimal tuning in presence of delays

In order to remediate the detrimental effects of time delays in the control loops, the PID gains can be optimally tuned based on the estimated time delays. Fig. 1(c) shows the time responses of the system for the same values of time delays shown in Fig. 1(b). However, in these cases the PID gains were re-tuned ($G_p^r = 81214$, $G_d^r = 0.5154$, $G_p^i = 0.0177$, $G_d^i = 0.000603$) based on the time delay present in the system instead of keeping the PID gains obtained for the ideal no-delay case. From this simulation study it is possible to conclude that optimal tuning of PID gains arises as an effective method to cope with the time delays. However, it is possible to note by comparing both time responses in Fig. 1(a) and (c) that the tracking quality of the controllers deteriorates as the time delay increases regardless of the optimal setting of the PID gains. This implies that in terms of performance there is a practical limit of time delays that well-tuned PID controllers can handle.

5. Stability analysis of the system

To prove stability of the time-delayed closed-loop system, we use the dual-locus method. The dual-locus technique, an extension of the Nyquist diagram technique, was originally proposed by

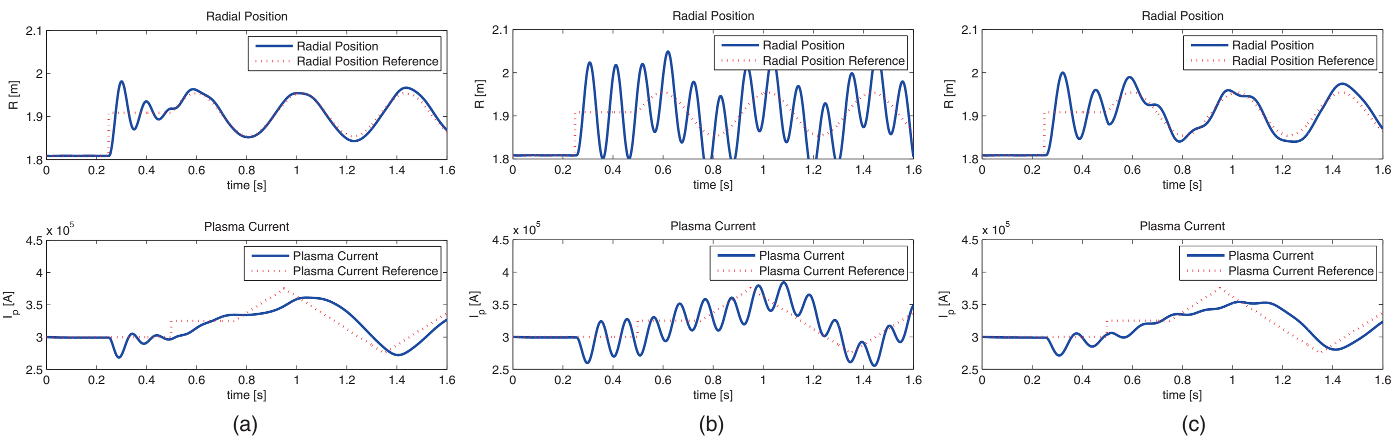

Fig. 1. Closed-loop time response with PID gains tuned by extremum seeking: (a) $\tau_o = 0$ with PID gains tuned for $\tau_o = 0$, (b) $\tau_o = 5$ ms with PID gains tuned for $\tau_o = 0$, (c) $\tau_o = 5$ ms with PID gains re-tuned for $\tau_o = 5$ ms.

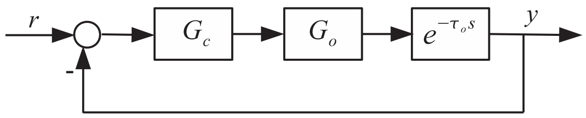

Fig. 2. Closed-loop time-delayed system.

Satche [3]. Assuming perfect decoupling, both current and radial position control loops can be represented as in Fig. 2, where G_o is the delay-free part of the plant, τ_o is the pure time delay of the plant and G_c is the controller. The closed-loop transfer function is given by

$$\frac{G(s)e^{-\tau_o s}}{1+G(s)e^{-\tau_o s}} \quad (1)$$

where $G(s)=G_o(s)G_c(s)$ is a stable transfer function. Stability of (1) has been extensively studied (see, e.g., [6]).

The characteristic equation of the closed loop system can be written as $F(s)=0$ with $F(s)=e^{\tau_o s}+G(s)$. Let Γ_c be the Nyquist contour and thus enclose the entire right half of the s-plane with the exception of singularities on the imaginary axis. From the Argument Principle [7], and since $F(s)$ has no pole in the interior of Γ_c ($G(s)$ is stable), the closed-loop system is stable if and only if the variation of the argument of $F(s)=e^{\tau_o s}+G(s)$ is zero. To apply this stability criterion a plot of $F(s)$ is needed, which requires the addition of $G(s)$ and $e^{\tau_o s}$. To avoid the summation of these two frequency-dependent functions, we plot separately $G(s)$ and $-e^{\tau_o s}$ (dual-locus). The Nyquist plot of $-e^{\tau_o s}$ is always a counterclockwise unit circle starting at the point $-1+j0$ for $\omega=0$. Fig. 3 shows a zoom around the origin of the dual-locus for both the radial position and plasma current loops. Noting that the characteristic equation can be rewritten as $G(s)=-e^{\tau_o s}$, the stability criterion can be evaluated from the dual-locus.

The existence of an enclosure of the origin by $F(s)$ (or alternatively of $-e^{\tau_o s}$ by $G(s)$), and therefore the stability of the system, can be evaluated from the dual-locus using the difference vector and frequency distributions techniques [8]. Under the condition that $n>m$ and $||a_m/b_n||<1$, where m and n denote the degrees of numerator and denominator of $G(s)$, and a_m and b_n denote the leading coefficients of the numerator and denominator, the stability criterion implies that the closed-loop

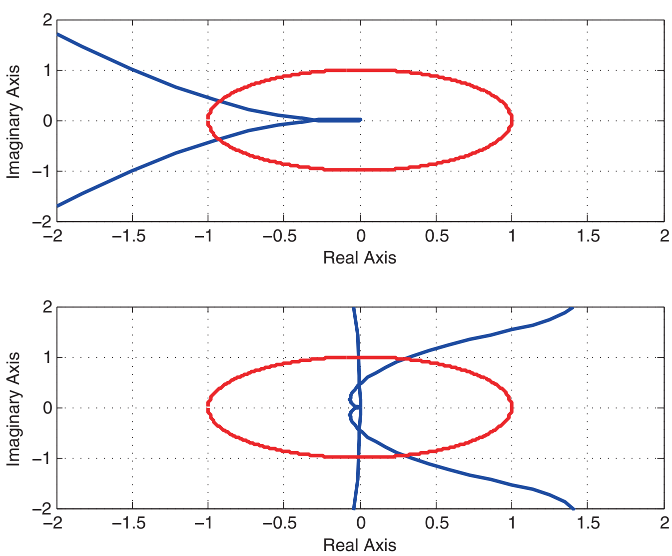

Fig. 3. Dual-locus ($G(s)$: blue, $-e^{\tau_o s}$: red): R loop (top), Ip loop (bottom). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

is stable if one of the following conditions holds [9]: (i) The equation $||G(j\omega)||=1$ has no positive real roots; (ii) The equation $||G(j\omega)||=1$ has one positive real root at $\omega=\omega_c$, and the inequality $-\pi+\tau_o\omega_c<arg[G(j\omega_c)]$ holds; (iii) The equation $||G(j\omega)||=1$ has two positive real roots at the frequencies ω_{c1} and ω_{c2} ($\omega_{c1}<\omega_{c2}$), and $-\pi+\tau_o\omega_{c2}<arg[G(j\omega_{c2})]$ or $\frac{1}{\omega_{c1}}\{arg[G(j\omega_{c1})]+(2k+1)\pi\}<\tau_o<\frac{1}{\omega_{c2}}\{arg[G(j\omega_{c2})]+(2k+3)\pi\}$. Here $k=0, 1, 2, ..., p$, where p is the maximal positive integer that makes the right term larger than the left one. In addition, the argument function $\arg(\cdot)\in[-\pi,\pi)$ by convention.

It can be noted from Fig. 3 that the Nyquist plot of the radial position delay-free loop gain crosses the unity circle at frequency $\omega_c=59.682$ rad/s with crossing point $(-0.928, -0.378)$ and the Nyquist plot of the plasma current delay-free loop gain crosses the unit circle at frequencies $\omega_{c1}=1.61\times10^{-10}$ rad/s and $\omega_{c2}=5.3160$ rad/s with crossing points $(0.00244, -1)$ and $(0.2989, -0.9543)$, respectively. By using the stability condition (ii) for the radial-position loop and (iii) for the plasma-current loop, it can be easily concluded that the whole system is stable if $\tau_o<6.4$ ms. Simulations show consistency with the obtained stability condition.

6. Modified Smith predictor

The Smith predictor (SP) is well known as a practical control method for systems with pure time delays. The main advantage of this method is that the time delay can be effectively taken outside the feedback look if the plant model is perfectly known. Fig. 4 shows the structure of a Smith predictor, where G_m is the model for the delay-free part of the plant and τ_m is the model for the pure time delay of the plant. The closed-loop transfer function of the augmented system in Fig. 4 is given by

$$\frac{y}{r}=\frac{G_c(s)G_o(s)e^{-\tau_o s}}{1+G_c(s)G_m(s)+G_c(s)G_o(s)e^{-\tau_o s}-G_c(s)G_m(s)e^{-\tau_m s}} \quad (2)$$

When the plant is perfectly known, i.e., $G_m\equiv G_o$ and $\tau_m=\tau_o$,

$$\frac{y}{r}=\frac{G_c(s)G_o(s)}{1+G_c(s)G_o(s)}e^{-\tau_o s} \quad (3)$$

which is the transfer function for the delay-free closed-loop system $(G(s)/(1+G(s)))$ multiplied by a pure time delay. Therefore, although there will still be a time delay in the response of the system, the delay-free response will be preserved.

Fig. 5(a) compares: (i) the time response of the delay-free case (solid blue), which has been also shown in Fig. 1(a); (ii) the time response of the $\tau_o=5$ ms case when the PID gains are those tuned for the delay-free case (dashed-dotted magenta), which has been also shown in Fig. 1(b); (iii) the time response of the $\tau_o=5$ ms case when the Smith predictor is implemented and the PID gains are still those tuned for the delay-free case (dashed green). At it is expected, since the time delay is only 5 ms, the difference between the free-delay (solid blue) and Smith-predictor (dashed green) cases is barely noticeable. By comparing Fig. 5(a) with Fig. 1(c), the predictor augmentation approach seems to be more effective than the optimal tuning approach. Fig. 5(b) shows the performance of the Smith predictor when the model of the plant is not perfectly known. We

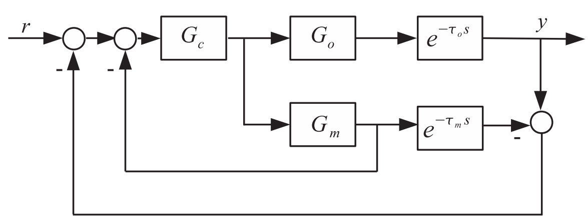

Fig. 4. Smith predictor structure.

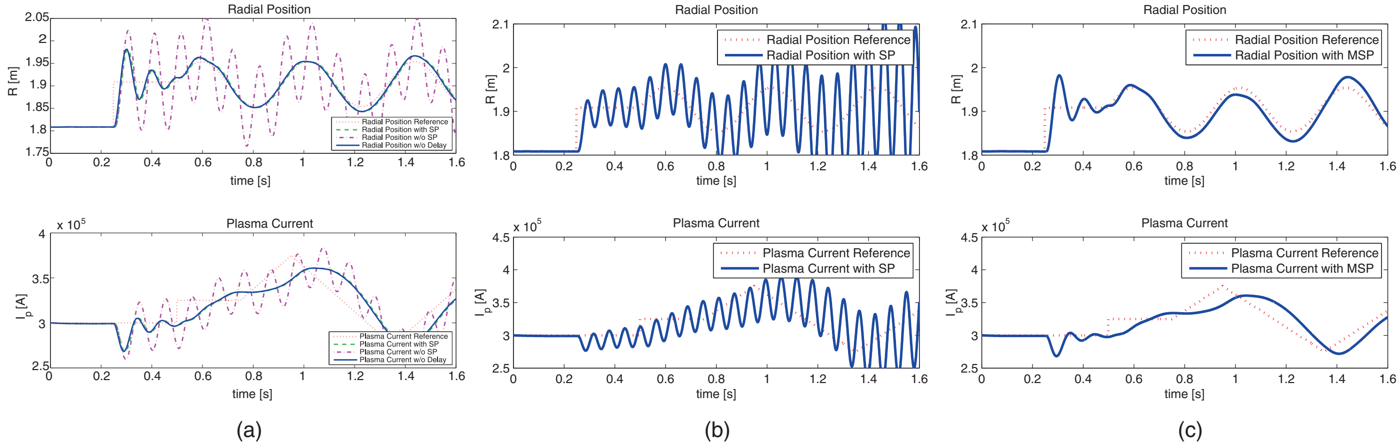

Fig. 5. Closed-loop time response for $\tau_o = 5$ ms with PID gains tuned by extremum seeking for the delay-free case: (a) Smith predictor with $\tau_m = 5$ ms, (b) Smith predictor with $\tau_m = 25$ ms, (c) modified Smith predictor with $\tau_m = 25$ ms.

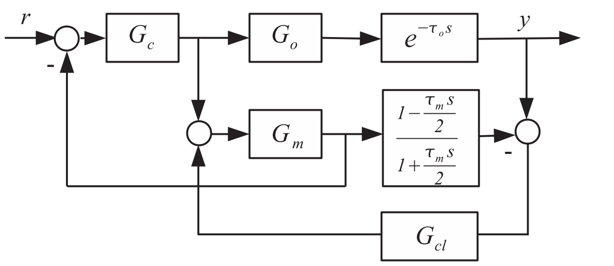

Fig. 6. Modified Smith predictor structure.

consider the case when $G_m = G_o$ but $\tau_m \neq \tau_o$. We notice from the figure that as τ_m departs from $\tau_o = 5$ ms, the performance of the Smith-predictor-based closed-loop system degrades.

To overcome this limitation, inspired by [10] we propose the modified Smith predictor (MSP) structure shown in Fig. 6. The controller G_{cl} is designed to make the difference between actual and predicted output converge to zero. The term $(1 - \tau_m s/2)/(1 + \tau_m s/2)$, denoted as G_d below, represents a Pade approximation for the pure time delay $e^{-s\tau_m}$. Fig. 5(c) shows the performance of the modified Smith predictor (G_{cl} was designed as a proportional controller) for the same cases presented in Fig. 5(b). The tracking performance is recovered even when the actual plant time delay is not well known.

To prove the stability of the proposed modified-Smith-predictor-based closed-loop system, we use the dual-locus technique introduced in Section 5. The closed-loop transfer function of the system in Fig. 6 can be written as

$$\frac{y}{r} = \frac{G_c G_o e^{-\tau_o s}(1 + G_{cl} G_m G_d)}{1 + G_{cl} G_m G_d + G_c G_m + G_c G_m G_{cl} G_o e^{-\tau_o s}} \tag{4}$$

and the characteristic equation can be written as

$$-e^{\tau_o s} = \frac{G_c G_m G_{cl} G_o}{1 + G_{cl} G_m G_d + G_c G_m} \tag{5}$$

By using the dual-locus stability conditions we conclude that the system stability condition is given by $\tau_o < 4.5967$ s.

7. Conclusions

Critical delay/lag values in the superconducting KSTAR tokamak were provided. PID-based controllers were designed using the concept of ohmic flux, which ideally decouples the current and position control loops. Simulation studies showed that optimally tuned PID controllers can successfully handle significant amounts of time delay. However, in terms of performance there is a practical limit of time delays that well-tuned PID controllers can handle. Beyond this limit an augmentation of the control loop by a novel predictor was proposed to improve the performance of the closed-loop system, i.e., to handle larger values of unknown time delays without deterioration of the tracking performance. It was shown that the proposed predictor is robust against uncertainties in the values of the delays. The dual-locus technique based on the Argument Principle was employed to assess stability in the presence of time delays of both the original and the augmented (by the predictor) systems.

Acknowledgements

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References

[1] K. Kim, H.K. Park, K.R. Park, B.S. Lim, S.I. Lee, Y. Chu, et al., Status of the KSTAR superconducting magnet system development, Nuclear Fusion 45 (2005) 783–789.
[2] N. Killingsworth, M. Krstic, PID tuning using extremum seeking, IEEE Control Systems Magazine 26 (1) (2006) 70–79.
[3] M. Satche, Discussion on stabilityof linear oscillating systems with constant time lag, Journal of Applied Mechanics, Transactions of the ASME 16 (4) (1949) 419–420.
[4] M.L. Walker, D.A. Humphreys, Valid coordinate systems for linearized plasma shape response models in tokamaks, Fusion Science and Technology 50 (2006) 473–489.
[5] M.L. Walker, Calculation of feedforward current and voltage trajectories for shape and plasma current control, Engineering physics memo epm060508a, General Atomics (2006).
[6] K.Gu, V.L. Kharitonov, J. Chen, Stability of Time-Delay Systems, Birkhä user, 2003.
[7] R.V. Churchill, Complex Variables and Applications, McGraw-Hill Book Company, Inc., New York, 1960.
[8] P.N. Nikiforuk, D.D.G. Nunweiler, Dual-locus stability analysis, International Journal of Control 1 (2) (1965) 157–166.
[9] L. Ou, W. Zhang, D. Gu, Nominal and robust stability regions of optimization-based PID controllers, ISA Transactions 45 (3) (2006) 361–371.
[10] S. Wang, B. Xu, Q. Wang, Y. H. Liu, Modified Smith predictor and controller for time-delay process with uncertainty, Proceedings of the 6th World Congress on Intelligent Control and Automation, June 21-23, 2006, Dalian, China, (1) 623-627.