TOWARDS DENSITY PROFILE REGULATION VIA PELLET INJECTION IN TOKAMAKS USING HYBRID MODEL PREDICTIVE CONTROL

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Abstract

A hybrid model predictive control (MPC) strategy accounting for the discrete-time nature of pellet injection is developed to actively regulate the plasma-density profile in a cylindrical geometry. Pellet injection is expected to be the primary fueling system of next-generation tokamaks such as ITER. This is due to its ability to overcome the limitations associated with gas puffing, such as the time delay in actuation caused by the machine's size and the inability of the neutral fueling gas to penetrate the plasma core. In this work, the discrete-time nature of pellet injection is taken into account during the synthesis of the density-profile controller. The pellet-injection system dynamics is approximated by a simple discrete-time model that intends to reproduce the fueling effects of the injected pellets in tokamaks like ITER. The density-profile evolution model, given by a one-dimensional partial differential equation, is discretized in space and time to reduce it into a set of difference equations to enable MPC design. The MPC scheme is designed by solving a finite-horizon optimization control problem that incorporates constraints on both the plasma state and the control inputs. Simulation studies are conducted to analyze the controller's performance.

1. INTRODUCTION

One of the main challenges in tokamak operation is maintaining accurate control of the plasma state [1] in order to preserve magnetohydrodynamic (MHD) stability and achieve the desired level of performance within safety limits. Active regulation of the plasma density is one of the several control problems that must be solved to achieve this goal. Existing tokamaks rely primarily on gas puffing to regulate the plasma density. Proportional-Integral-Derivative (PID) controllers actuate gas valves based on the deviation between requested and measured plasma density [2]. However, fueling via gas puffing may not be sufficient in next-generation tokamaks like ITER because of the inability of the particles injected by gas puffing to penetrate the plasma since temperatures and densities are high at the core. In addition, since the distance between the plasma vessel and the gas puffing valves is significant, particle fueling via gas puffing systems in large tokamaks is characterized by important time delays. As a result, it can be challenging to regulate the core density by using only gas puffing in reactor-grade tokamaks. Another technique that can be used for plasma fueling is pellet injection, which is achieved by firing pellets of frozen fuel into the plasma [3] to fuel the core efficiently. Pellet injection could achieve deep core fueling and is expected to be ITER's primary fueling system [4]. As illustrated in Fig. 1, ITER is set to employ pellet injection from both the high-field side (left in the figure) and the low-field side (right in the figure).

In tokamaks, not only the volume-averaged density (whose dynamics is modeled by a nonlinear ordinary differential equation (ODE)) but also the entire profile (whose dynamics is modeled by a nonlinear partial differential equation (PDE)) may need to be precisely controlled in real time. The problem of density profile control has been tackled in [5, 6, 7] by using backstepping techniques for both non-burning and burning plasmas in a one-dimensional (1D) cylindrical geometry. The use of a cylindrical geometry is a first approach to the density control problem as the cylindrical symmetry simplifies mathematical modeling and analysis. More recently, robust control [8, 9, 10] and model predictive control (MPC) [11] techniques have been exploited to approach the problem of density-profile control [12] in tokamaks. Most existing density-profile control solutions currently treat actuation requests to pellet injection systems as continuous-time variables. However, it is essential to recognize that pellet injection exhibits

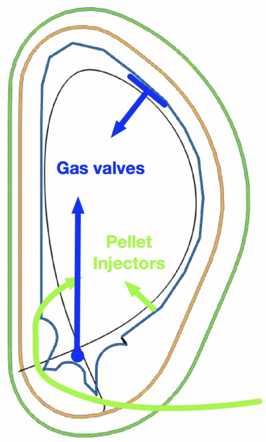


FIG. 1. Poloidal cross-section of ITER identifying pellet injectors and gas valves that could be used for fueling.

discrete-time behavior, which can result in localized density fluctuations. These variations in the plasma state may provoke unanticipated responses from the feedback controller. Even if the controller is designed to be robust to such changes in the state, the conversion of the continuous-time pellet-injection request into discrete-time actuation can cause deviation from the desired density targets. To address this challenge, there is a pressing need to develop response models and subsequent model-based control solutions that incorporate the discrete behavior of pellets. Such response models will allow for more accurate simulations to assess expected closed-loop performance in real tokamaks, while such controllers will enable tighter regulation of the density profiles around their desired targets in experimental plasma discharges.

A mixed-integer MPC that prescribes binary pellet injection input values, corresponding to "pellet fired" and "no pellet fired" states of the pellet injection system, was recently proposed in [13]. In this work, a hybrid MPC scheme accounting for the discrete nature of pellet injection is proposed. The MPC scheme is hybrid in the sense that it incorporates both continuous and discrete constraints. In particular, the evolution of the state, which is the plasma density, is governed by a continuous-time linear differential equation, whereas, the input can only assume discrete values. The proposed MPC is based on a linear control-oriented response model in 1D cylindrical geometry whose actuator model implicitly incorporates two distinct phases (see Section 2) of pellet injection. The control scheme, by formulation, determines whether or not to inject a pellet at a given time instant of the plasma discharge. Thus, the actuator request prescribed by the MPC does not need conversion as in the case of controllers that prescribe continuous actuator requests. Furthermore, the proposed control scheme is computationally efficient since the optimization process in MPC only entails selecting between two possible actions of either injecting a pellet or withholding it. Numerical simulations are used to demonstrate the effectiveness of the proposed MPC scheme.

The remainder of this paper is organized as follows. The model for density profile evolution, incorporating the discrete nature of the pellets, is presented in Section 2. Section 2 also explains the steps involved in simplifying the original model to a form that is suitable for MPC. The hybrid MPC algorithm based on the model derived in Section 2 is presented in Section 3. Simulations studying the performance of the proposed algorithm are provided in Section 4. Finally, concluding remarks and potential future research are discussed in Section 5.

2. MODEL FOR DENSITY PROFILE EVOLUTION USING PELLET INJECTION

The 1-D model used in this work to represent the dynamics of the density profile is based on an ion transport PDE in cylindrical coordinates [6], which is given by

$$\frac{\partial n}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r\left(D\frac{\partial n}{\partial r} - nV_p\right) + S, \tag{1}$$

subject to the boundary conditions

$$\left.\frac{\partial n}{\partial r}\right|_{r=0} = 0, \quad \left.\frac{\partial n}{\partial r}\right|_{r=a} = k_n n(a), \tag{2}$$

where $n(r,t)$ is the density at location $r \in [0,a]$ and time t, D is the diffusion coefficient, V_p is the inward pinch velocity, and k_n is a constant. The term $S(r)$ accounts for the particle injection rate corresponding to the pellet injection systems. It is important to note that the pellets injected into the plasma exhibit a discrete-time behavior, which can be classified into two phases. In the first phase, an injected pellet enters the plasma and gradually transfers particles. After all the particles from the pellet have been transferred to the plasma, there is an idle time until the next pellet injection. This time interval constitutes the second phase. The source model must incorporate these two phases to account for the discrete-time behavior of the pellets. In the following analysis, the first and second phases are assumed to be of equal duration. Then, the source term S is given by

$$S = \sum_{j=1}^{g} S_j(r)u_j(t), \tag{3}$$

$$u_j(t) = \begin{cases} \gamma, \; t \in [t_{2m}, t_{2m+1}) \\ 0, \; t \in [t_{2m+1}, t_{2m+2}) \end{cases}, \qquad \gamma \in \{0,1\}. \tag{4}$$

In the above source model, γ determines the on and off action of each pellet injection system, $u_j(t)$ is the controlled input capturing the discrete nature of the actuators (pellet injectors), g is the number of pellet injectors, and $S_j(r)$ describes the deposition profile of the actuators. Moreover, the term t_{2m} is defined as $t_{2m} = t_0 + 2mT$, where T represents the duration of the first and second phases, $m \in \{0,1,2,3,\ldots\}$ is pellet injection cycle index, and t_0 is the pellet injection start time. During the interval $[t_{2m}, t_{2m+1})$, the pellet injection system can either inject a pellet or take no action. This interval corresponds to the first phase described above. However, during the subsequent interval $[t_{2m+1}, t_{2m+2})$, no pellet is injected, irrespective of the action taken at the previous time interval, which corresponds to the second phase. Thus, this model inherently accounts for the time gap between two pellets injected into the plasma. As mentioned earlier, the idle second phase is assumed to be equal to the time taken by each pellet to inject particles into the plasma. However, this model can be generalized to unequal time intervals to account for more general cases and the proposed MPC can be easily adapted to such cases. The deposition profiles S_j play a key role in understanding the behavior and the impact of the pellets injected into the plasma. The deposition profile describes the mass and the size of the injected pellets. The size of the pellet is determined by the shape of the deposition profile, and the mass of pellet is determined by the area under the profile.

The model given in (1) is a partial differential equation (PDE). Since a reduced-order model is needed for the synthesis of the feedback controller, (1) is spatially discretized. Discretizing the model in (1) at $l+1$ equidistant nodes in the spatial interval $[0,a]$ using the finite difference method (FDM) results in an ordinary differential equation (ODE) of the form

$$\dot{x} = \alpha x + \beta u, \tag{5}$$

$$\begin{aligned}
\alpha &= \begin{bmatrix} a_1+b_1 & c_1 & & & \\ a_2 & b_2 & c_2 & & \\ & \ddots & \ddots & \ddots & \\ & & a_{l-2} & b_{l-2} & c_{l-2} \\ & & & a_{l-1} & b_{l-1}+\frac{c_{l-1}}{1-hk_n} \end{bmatrix}, \qquad \beta = \begin{bmatrix} S_{1,1} & \cdots & S_{1,g} \\ S_{2,1} & \cdots & S_{2,g} \\ \vdots & \vdots & \vdots \\ S_{l-2,1} & \cdots & S_{l-2,g} \\ S_{l-1,1} & \cdots & S_{l-1,g} \end{bmatrix}, \\
a_j &= \frac{V_p}{2h} + \frac{D}{h^2} - \frac{D}{2jh^2}, \qquad b_j = -\frac{V_p}{jh} - \frac{2D}{h^2}, \\
c_j &= \frac{D}{h^2} + \frac{D}{2jh^2} - \frac{V_p}{2h}, \qquad j = 1,\ldots,l-1,
\end{aligned} \tag{6}$$

where the state $x = [n_1, \cdots, n_{l-1}]^T$ represents the vector of densities at the interior finite-difference nodes r_j for $j = 1,\ldots,l-1$, $u(t) = [u_1(t), \cdots, u_g(t)]^T$ is the input, $h = a/l$ is the distance between two adjacent nodes, and the location of the the finite-difference nodes is given by $r_i = ih$ for $i = 0,\ldots,l$ (including boundary points).

Model predictive control algorithms need discrete-time models to enable practical implementation in the plasma control system. Thus, a temporal discretization of (5) using a zero-order hold discretization with period T, the duration of the first and second phases, is carried out. If $t_k = kT$ represents the k^{th} time step, then the resulting discrete-time model is written as

$$\begin{aligned} x(t_{k+1}) &= Ax(t_k) + Bu(t_k), \\ A = e^{\alpha T}&, B = \int_0^T e^{\alpha\tau} d\tau \beta, \end{aligned} \tag{7}$$

with the input $u(t_k) = [u_1(t_k), \cdots, u_g(t_k)]^T$ for

$$u_j(t_k) = \begin{cases} \gamma, & \text{if } k \text{ is even,} \\ 0, & \text{if } k \text{ is odd.} \end{cases} \tag{8}$$

Note that since the temporal discretization is carried out with period T, the even and odd conditions on the time index k correspond to the first and second phases of the plasma injection cycle described above, respectively.

3. CONTROL SYNTHESIS

This section reviews the hybrid MPC algorithm, which is a control strategy that uses the system model to predict its future behavior, and optimize the control inputs over a finite time horizon while accounting for state and input constraints. The general procedure for implementing state-feedback MPC algorithms in digital environments can be described as follows:

(i) At time t_k, the system samples the state $x(t_k)$ and transfers it to the controller.

(ii) The controller solves an N-step discrete-time finite horizon optimal control problem (FHOCP) with the initial state $x(t_k)$. The result of FHOCP is a vector of N inputs each corresponding to the time-steps in the finite horizon.

(iii) The input value corresponding to the first time step of the finite horizon is implemented in the system.

(iv) At the next time step t_{k+1} the whole process is repeated again.

As evident from the above decription, solving an FHOCP is a critical step in implementing MPC. The FHOCP can be formulated as follows. At time step t_k

$$\min_{u^{k,i}, i=0,\cdots,N-1} \quad J[u^{k,i}|x(t_k)] \tag{9a}$$

$$J[u^{k,i}|x(t_k)] = \sum_{i=0}^{N-1} \Big((x^{k,i} - \bar{n})^T Q (x^{k,i} - \bar{n}) + (u^{k,i})^T R u^{k,i} \Big) + (x^{k,N} - \bar{n})^T P (x^{k,N} - \bar{n}), \tag{9b}$$

$$\text{such that} \quad x^{k,i+1} = A x^{k,i} + B u^{k,i}, \; x^{k,0} = x(t_k), \tag{9c}$$

$$u_j^{k,i} = \begin{cases} \gamma, & \text{if } k+i \text{ is even,} \\ 0, & \text{if } k+i \text{ is odd,} \end{cases} \tag{9d}$$

$$\gamma \in \{0,1\}, \tag{9e}$$

where $J[u^{k,i}|x(t_k)]$ is the cost function, k is the integer that corresponds to the time step t_k, N is the prediction horizon, $\bar{n}$ is the target density, Q, P and R are positive symmetric weighting matrices, and A, B are determined by (7). The terms $x^{k,i}$ and $u^{k,i}$ are the prediction state and input at the i^{th} time-step in the N-step horizon, respectively. The hybrid nature of the MPC arises from the fact that the input takes discrete values. The state evolution is continuous and smooth when the input is 0. However, when a pellet is injected, the instantaneous change in input can make the density increase suddenly as illustrated in Fig. 2, which shows the results of numerical simulations discussed in Section 4.

The plasma control system (PCS) implementation of the hybrid MPC requires solving the above formulated FHOCP at each time step. Dynamical programming packages or other optimization tools can be used to solve the FHOCP. However, since the FHOCP presented above involves choosing between two possible values of the input at each time step, a much simpler algorithm that can generate the solution of the FHOCP can be devised. At time step t_k, if the index k is even, the FHOCP is solved by executing the following steps:

(i) For a given horizon length N, create an array of possible value of the input $u^{k,i}$ for $i = 0, \ldots, N-1$. For instance, if the horizon length is 6 steps and one pellet injector exists, then input combination U_C is

$$U_C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}, \tag{10}$$

where each row represents the possible combinations of input actions that the controller can take in an N-step horizon.

(ii) For each possible combination of the input sequence, compute the sequence of state values $x^{k,i+1}$ using (9c) and compute the cost function value using (9b).

(iii) Choose the input sequence corresponding to the least cost function value.

The input value corresponding to the first time step of the optimal input sequence obtained from matrix (10) is used as the control action. At time steps with odd k values, the input is automatically set to 0 without any further computation. These above set of actions are faster than conventional optimization schemes since they implicitly exploit the constraints of the problem.

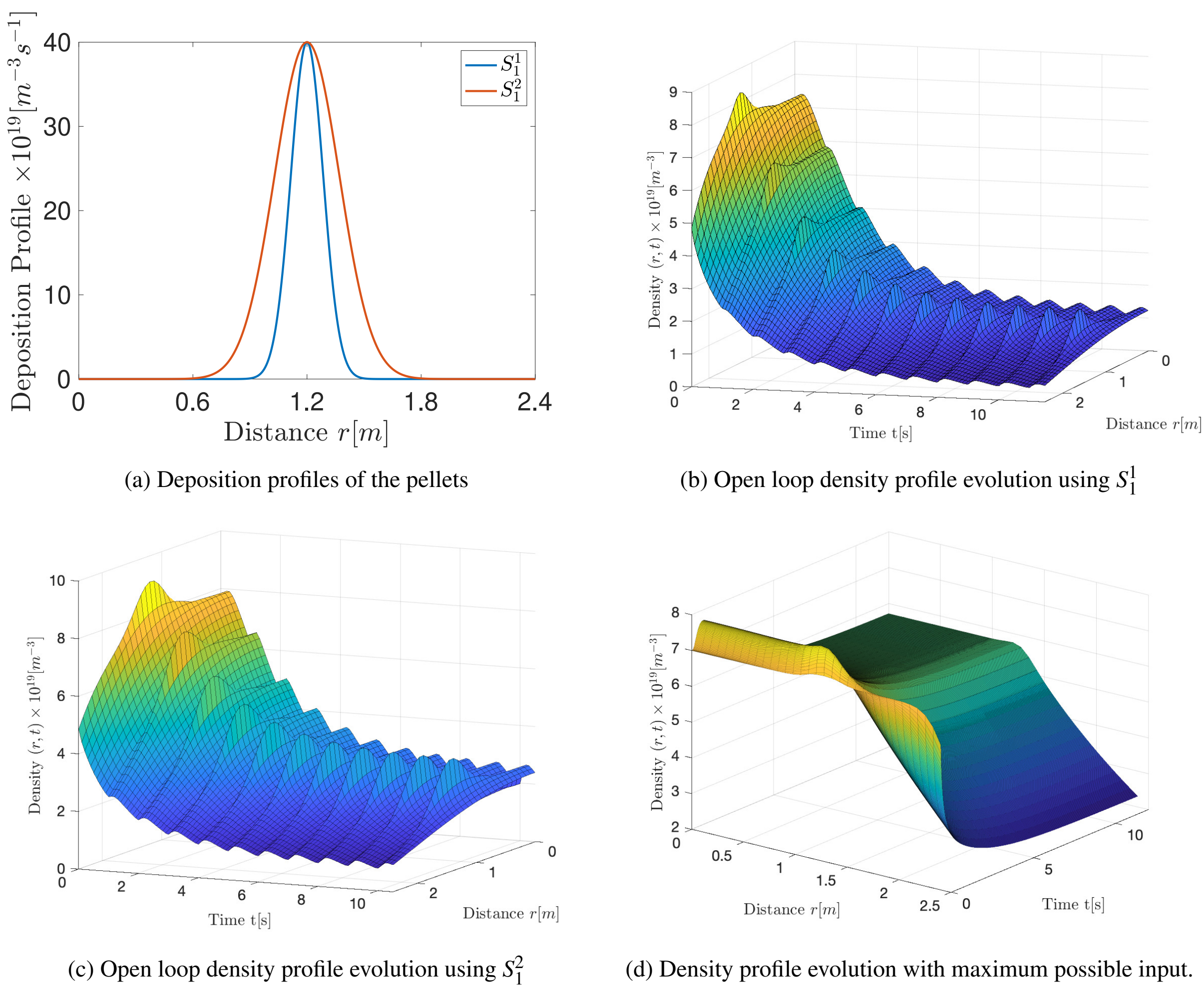
4. SIMULATION STUDY

The following simulation studies assess the performance of the proposed hybrid MPC scheme for density profile regulation. In the simulations, the sampling interval and the length of the prediction horizon are set as $T = 0.01$ seconds and 6 time steps, respectively, and one pellet injection system is assumed to be available for control, i.e., $g = 1$. The specific values used in the simulations for each parameter in the model are provided in the table below.

$a(m)$	k_n	$T(s)$	D	V_p	$n(t=0, r \in [0,a])(m^{-3})$	$S_1^1(\times 10^{20} m^{-3} \cdot s^{-1})$	$S_1^2(\times 10^{20} m^{-3} \cdot s^{-1})$
2.4	-1	0.01	1	0	7×10^{19}	$4e^{-\frac{(0.5-(r-1)h/a)^2}{0.0025}}$	$0.895e^{-\frac{(0.5-(r-1)h/a)^2}{0.05}}$

Table 1: Model parameters and initial conditions of the density model

As shown in the table, the actuator deposition profiles, denoted as S_1^1 and S_1^2, are modeled as Gaussian radial basis functions and are illustrated in Fig. 2a. Note that the area under the profiles and their width correspond to the mass and the size of the pellet, respectively. Two open loop simulations were conducted. Both simulations shared identical initial conditions and used an input sequence with a pellet injected every 1 second for a total duration of 11 seconds. Both Fig. 2b and Fig. 2c show the influence of the pellets on the density profile. The moment the pellets are injected, an instantaneous spike of density occurs. As expected, even though the same number of pellets was injected, the evolution of the density profiles in the two cases is different since the total number of particles is different. Furthermore, it is evident from these two figures that the effect on the density in the case of S_1^1 is more localized while compared to the case of S_1^2.



(a) Deposition profiles of the pellets

(b) Open loop density profile evolution using S_1^1

(c) Open loop density profile evolution using S_1^2

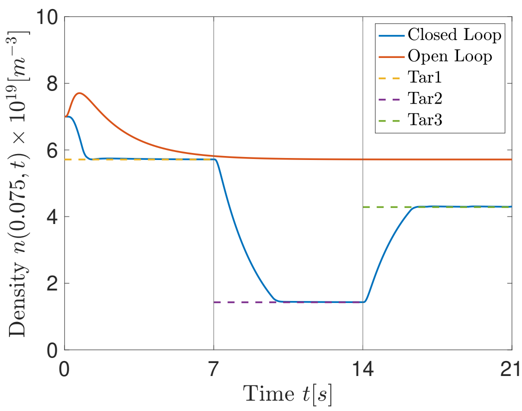
(d) Density profile evolution with maximum possible input.

FIG. 2. The density and deposition profiles of the open loop simulations

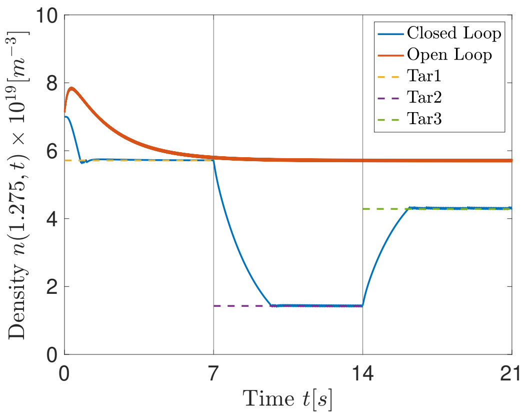
To analyze the range of feasible controller targets for the control algorithm, an open-loop simulation was carried out using the parameters presented in Table 1, the deposition profile S_1^1, and the "maximum possible input." Since T was selected to be 0.01 seconds, the term "maximum possible input" means that a pellet is injected every 0.02 seconds. Fig. 2d shows the density profile evolution obtained from the open-loop simulation. The figure demonstrates that the density profile stabilizes to a "steady-state-like" value after 11 seconds. Note that the continuous injection of the pellets causes localized fluctuations in the density; hence, the density profile never reaches a perfect steady-state value. The final density profile obtained from the simulation using the maximum possible input gives an upper bound on the density values the control algorithm can achieve. This simulation was the basis for selecting targets for the closed-loop simulations.

Two closed-loop simulation cases were considered to analyze the performance of the proposed hybrid MPC. The two cases differ in the choice of the initial density profile, open-loop inputs, and targets. Their results are shown in Fig. 3 and Fig. 4. In the figures, the red and blue lines represent the open-loop and closed-loop trajectories, respectively. The yellow-dashed lines denote Target 1, which matches the steady state values generated by the open loop simulation. The purple-dashed and green-dashed lines denote Target 2 and Target 3, respectively, which are arbitrary feasible values. In the closed-loop simulations, Target 1 was used between 0 and 7 seconds; Target 2 was used between 7 and 14 seconds; Target 3 was used for the rest of the simulation. Target 1, Target 2, and Target 3 are separated in the plots by distinct black lines corresponding to specific times.

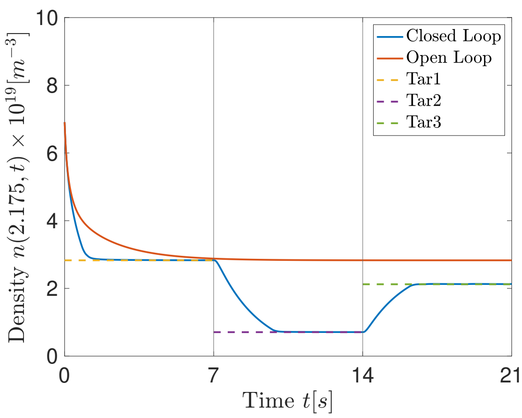
4.1. Case 1: Maximum Pellet Injection Sequence



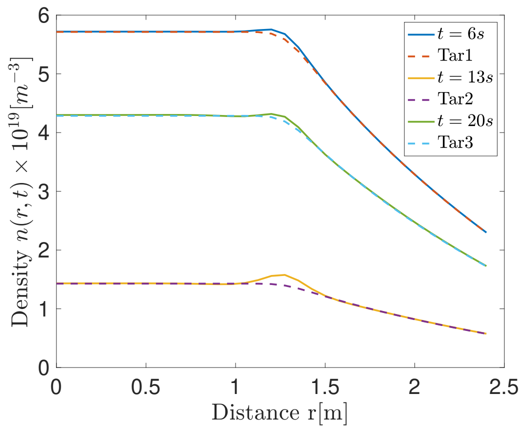
(a) The plasma density at $r = 0.075m$



(b) The plasma density at $r = 1.275m$



(c) The plasma density at $r = 2.175m$



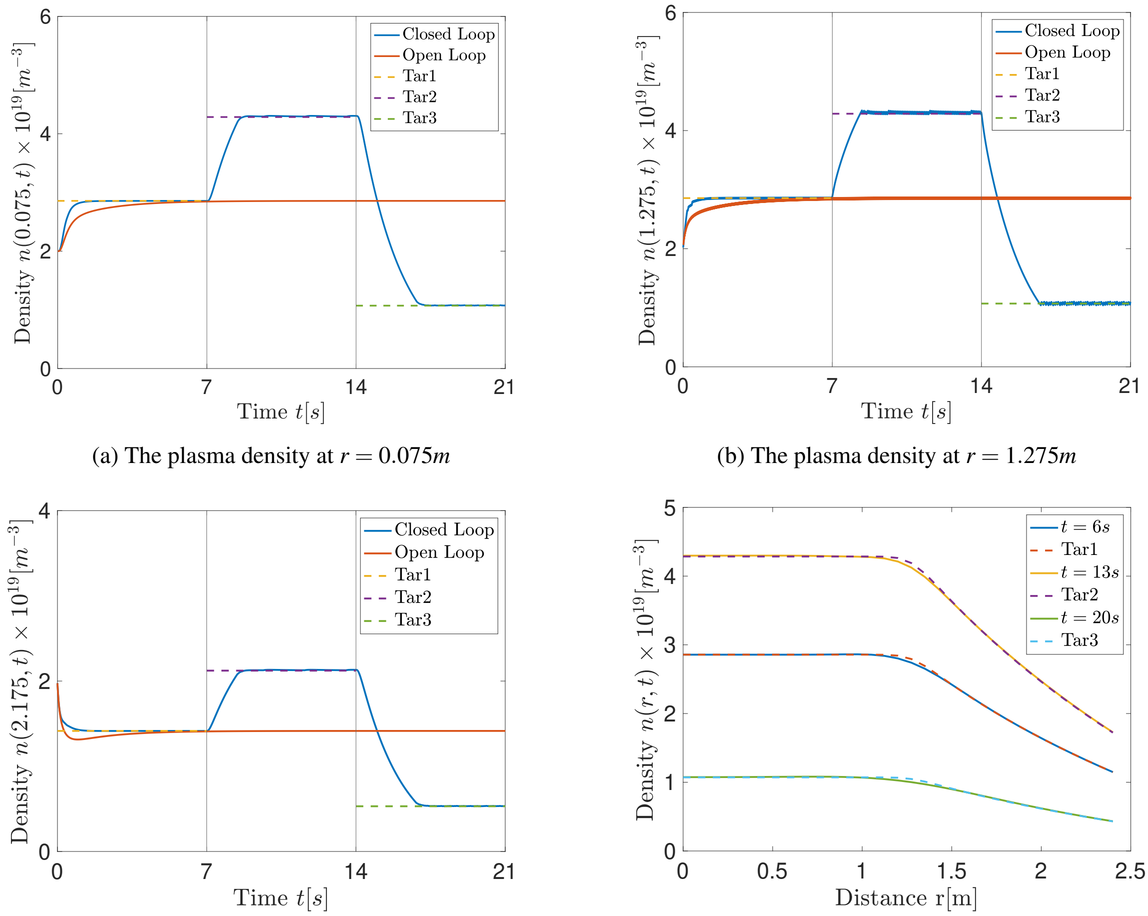
(d) The plasma density at $r = 2.175m$

FIG. 3. Case 1: Time evolution of density profile at different locations.

Fig. 3 shows the open-loop and closed-loop simulation results of this case. The open loop simulation uses the same maximal pellet injection sequence that is used to create Fig. 2d. From the onset of the simulations, it is clear that their evolution have vivid differences. Target 1 holds particular significance as it aligns with the steady-state density value observed in the open-loop simulation. The purpose of Target 1 is to examine the controller's capability of converging to steady state. The controller achieves the target within 2 seconds of the simulation, which represents a faster convergence. Targets 2 and 3 have been selected to evaluate the controller's ability to track densities distinct from the steady state. Both targets are achieved within few seconds of activating them.

4.2. Case 2: Non-Maximum Pellet Injection Sequence

The results for the second test case are shown in Fig. 4. As previously noted, this case employs distinct targets and an initial density profile in contrast to the first case. Similar to the first case, Target 1 corresponds to the "steady-state-like" values obtained from the open-loop simulations. However, a pre-defined non-maximal pellet injection sequence was used in this case. Specifically, a pellet was injected every 0.04 seconds for the open loop simulation.



(a) The plasma density at $r = 0.075m$

(b) The plasma density at $r = 1.275m$

(c) The plasma density at $r = 2.175m$

(d) The plasma density at $r = 2.175m$

FIG. 4. Case 2: Time evolution of density profile at different locations.

As in the first case, the hybrid MPC expedited the convergence of the density to Target 1. Furthermore, the MPC is able to effectively drive the density to the target whenever new targets are introduced.

5. CONCLUSION AND FUTURE WORK

A hybrid MPC strategy for the regulation of the density profile in a cylindrical plasma by pellet injection has been developed and tested using numerical simulations. The plasma-density evolution is modeled by a 1D transport equation and a source term that accounts for the discrete nature of the pellet. The 1D model is then reduced to a set of difference equations by spatial and temporal discretization to enable control design. The difference equations are then used to design a computationally efficient hybrid MPC algorithm. The simulation studies illustrate the effectiveness of the proposed MPC scheme in driving the density profile to a given target.

This work represents an initial step towards achieving effective density control in reactor-level tokamaks like ITER using algorithms that implicitly consider the discrete nature of pellets. A more realistic density-profile response model will be considered in the future by moving to a tokamak (toroidal) geometry, employing more sophisticated models for the diffusion coefficient D and pinch velocity V_p, and coupling the 1D model for the plasma core with 0D models of the scrape-off layer (SOL) and divertor regions. This will also enable the design of control schemes using different fueling sources and, therefore, complementing multiple pellet injection systems in the plasma core with gas puffing at the plasma boundary. Since the 1D model for the core and the 0D models for the SOL/divertor regions are connected through density fluxes at the boundary, Neumann boundary conditions have been used in this work in anticipation of this future integration step. Moreover, the control-oriented model for pellet injection will be further developed to include the nonlinear behavior of the fueling efficiency as a function of the density and temperature conditions in the plasma and to develop adaptive schemes to cope with the resulting variability and uncertainty. This model will also allow for adaptation and optimization of the control scheme in terms of pellet size, pellet velocity, and injection frequency.

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DISCLAIMER

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