Antiwindup Scheme for Plasma Shape Control with Rate and Magnitude Actuation Constraints in the DIII-D Tokamak*

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***Abstract*— The Advanced Tokamak (AT) operating mode is the principal focus of the DIII-D tokamak. In order to be prepared for the higher control demands arising in AT scenarios, current efforts are focused on the development of an integrated multivariable controller to take into account the highly coupled influences of equilibrium shape, profile, and stability control. The first step of this project is the design of the shape and vertical controller which will be integrated in the future with control of plasma profiles. The time-scale properties of the system allow the separation of the vertical stabilization problem, approached by the authors in prior work, from the shape control problem, which is the focus of this paper.**

Ensured the vertical stability of the plasma, in this work we implement an anti-windup compensator that keeps the given nominal shape controller well-behaved in the presence of rate and magnitude constraints at the input of the nonlinear plant. The anti-windup synthesis problem is to find a nonlinear modification of the nominal linear controller that prevents undesirable oscillations but leaves the nominal closed loop unmodified when there is no input saturation.

I. INTRODUCTION

Demands for more varied shapes of the plasma and requirements for high performance regulation of the plasma boundary and internal profiles are the common denominator of the Advanced Tokamak (AT) operating mode in DIII-D [1]. This operating mode requires multivariable control techniques [2] to take into account the highly coupled influences of equilibrium shape, profile, and stability control. Current efforts are focused on providing improved control for ongoing experimental operations, preparing for future operational control needs, and making advances toward integrated control for Advanced Tokamak (AT) scenarios.

Vertical and Shape Control: The initial step toward integrating multiple individual controls is implementation of a multivariable shape and vertical controller for routine operational use which can be extended to integrate other controls. The long term goal is to integrate the shape and vertical control with control of plasma profiles such as pressure, radial E-field, and current profiles using feedback commands to actuators such as gas injectors, pumps, neutral beams (NB), electron cyclotron heating (ECH), and electron cyclotron current drive (ECCD).

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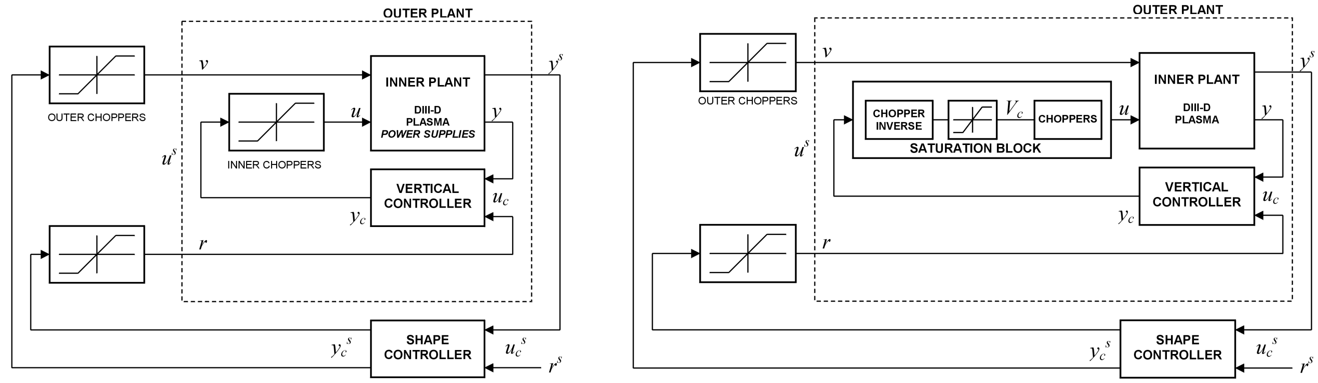
The problem of vertical and shape control in tokamaks was and is still extensively studied in the fusion community. A recent summary of the existing work in the field can be found in [3]. Several solutions for the design of the nominal controller were proposed for different tokamaks using varied control techniques based on linearized models. However, only a few of them [4] take into account the control voltage constraint in the design of the nominal controller.

Our approach is different in concept. The input constraints are not taken into account at the moment of designing the nominal controller. The goal is not the design of the nominal controller but the design of an anti-windup compensator that blends any given nominal controller, which is designed to fulfil some local (saturation is not considered) performance criterion, with a nonlinear feedback designed to guarantee stability in the presence of input saturation but not necessarily tuned for local performance. This nonlinear modification of the nominal controller also aims at keeping the nominal controller well-behaved and avoiding undesirable oscillations. The anti-windup augmentation must in addition leave the nominal closed loop unmodified when no saturation is present.

Shape Control Methodology: The shape control methodology at DIII-D is based on “isoflux control”. The isoflux control method, now in routine use on DIII-D, exploits the capability of the real time EFIT plasma shape reconstruction algorithm to calculate magnetic flux at specified locations within the tokamak vacuum vessel. Real time EFIT can calculate very accurately the value of flux in the vicinity of the plasma boundary. Thus, the controlled parameters are the values of flux at prespecified control points along with the X-point r and z position. By requiring that the flux at each control point be equal to the same constant value, the control forces the same flux contour to pass through all of these control points. By choosing this constant value equal to the flux at the X-point, this flux contour must be the last closed flux surface or separatrix. The desired separatrix location is specified by selecting one of a large number of control points along each of several control segments. An X-point control grid is used to assist in calculating the X-point location by providing detailed flux and field information at a number of closely spaced points in the vicinity of the X-point.



(a) Fig. 1. Nonlinear Power Supplies. (b)

Several problems make practical implementation of shape and vertical position controllers on DIII-D challenging: (P1) Computational speed is insufficient to do both vertical stabilization and shape control with the same controller. (P2) Vertical stability control and shape control share the same actuators. This is a particular problem for the outer coils because they become the only coils which perform shape control for the outer plasma boundary. (P3) The shape control power supply system is extremely nonlinear. (P4) Limitations on actuator voltage imply that commands to shaping power supplies (choppers) often saturate, particularly with large or fast disturbances.

The paper is organized as follow. Section 2 introduces the strategy used to face the challenges presented by the plasma shape and vertical position control problem. The contribution of this paper is clearly stated in this section. Section 3 explains the particular structure of the plant. Section 4 introduces the basis of the anti-windup method. The control design is explained in Section 5. The conclusions are finally presented in Section 6.

II. Control Strategy

Computational Speed Limitation: Time-scale separation of vertical and shape control appears to be critical for DIII-D, since multivariable shape controllers can require significant computation. Details of the architecture of the control design can be found in [5].

Actuator Sharing: A method implemented for sharing actuators involves constructing a linear controller which simultaneously stabilizes and provides control of vertical control coil currents on a fast time scale. The closed loop system comprised of the DIII-D plant (inner plant) and stabilizing vertical controller can act as an inner control loop for the shape control. The inner loop provides as input actuators the 6 vertical coil current and centroid vertical position reference signals r, and up to 12 shape coil command voltages v. By integrating control of the vertical control coils into a stabilizing controller, conflicts between shape and vertical control use of these coils is eliminated. “Frequency sharing” is accomplished explicitly with an H-infinity loop shaping design by weighting low frequencies to regulate the coil currents and high frequencies to stabilize the plasma. The design technique ensures that the overall system remains robustly stable.

Power Supply Nonlinearities: The problem of nonlinear choppers was addressed previously by constructing closed loop controllers for the outer chopper power supplies [6] using a nonlinear output inversion. However, this solution is not fast enough to be implemented in the inner loop. A possible approach to deal with the highly nonlinear inner chopper power supplies is shown in Figure 1-a. A linearized model of the choppers is incorporated to the inner plant that is used to synthesize the nominal linear vertical controller. The output of the controller represents in this case the command voltages to the choppers that saturate at fixed values (± 10 V). In this approach the nonlinear behavior of the power supplies are not considered and the linearized model of the plant is accurate only within a very restricted domain. To take into account the nonlinear nature of the power supplies we remove the linearized model of the choppers from the inner plant and incorporate a full nonlinear model of them into an augmented saturation block as it is shown in Figure 1-b. The nominal linear vertical controller is synthesized now using no information of the choppers and its output now represents directly the desired coil voltages. A chopper inverse function computes the necessary command voltages within the saturation levels to make the output voltage of the choppers equal to the desired coil voltages. When the chopper inverse funtion fails calculating those command voltages we say we have saturation. Although the saturation levels of the command voltages are still fixed values (± 10 V), the saturation levels of the augmented saturation block are now time-dependent functions (coil load current and DC supply voltage). Similarly, the saturation levels of the outer choppers also depend on the coil load currents and DC supply voltages.

Constrained Control: In order to make this approach succesful the inner controller (vertical controller) must guarantee the stability of the plant for all commands coming from the

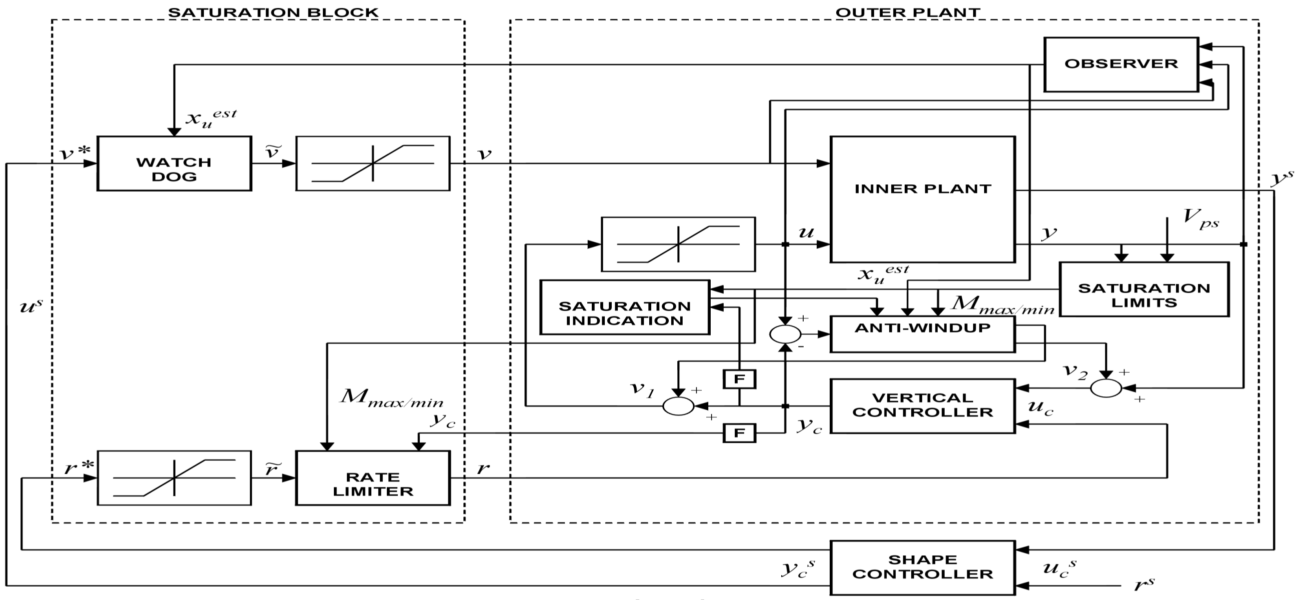


Fig. 2. Plant architecture.

outer controller (shape controller). However, the constraints on the input of the plant due to the saturation of the actuators may prevent this goal from being achieved. The saturation of the coil voltages can not only degrade the performance of the closed loop system but also impede the vertical stabilization when the synthesis of the nominal inner controller does not account for plant input saturation. Although the saturation of coil currents and voltages is a common problem in tokamaks and there are efforts to minimize the control demand for shape and vertical control and to avoid saturation [7], [8], the saturation of the actuators are generally not taken into account in the design of the nominal controllers in present works.

Recently several anti-windup approaches have been proposed [9], [10], [11], [12], [13], [14], [15]. Due to the characteristics of our problem we follow the ideas introduced in the companion papers [16], [17] and also discussed in [18] for exponentially unstable systems and [19] for nonlinear systems. This technique has been shown to be succesful in several case studies. However, the method must be modified and complemented in order to fulfill the performance requirements of our system.

The inner loop design must take care of the windup of that loop and ensure vertical stability for any command coming from the outer controller. We understand as windup the phenomenon characterized by degradation of nominal performance and even loss of stability due to magnitude and/or rate limits in the control actuaction devices. The anti-windup synthesis problem is to find a nonlinear modification of the predesigned nominal linear controller that prevents vertical instability and undesirable oscillations but leaves the nominal closed loop unmodified when there is no input saturation. This problem was approached in [20].

In this work we focus on the design of an anti-windup augmentation for the outer loop that prevents decrease of performance due to undesirable oscillations of the controller output. The main goal is to keep the shape controller well-behaved in presence of constraints at the input of the outer plant. The outer loop design must take care not only of the windup of the actuators of that loop but also of the input constraints imposed by physical limitations and the inner loop design mainly. Contrary to the inner loop design, where the anti-windup augmentation was designed for an exponentially unstable but linear plant, in this case we are dealing with a stable but nonlinear plant.

III. Plant Structure

The outer plant is the result of the anti-windup augmentation for the inner plant. The structure of our system is shown in figure 2. The inner plant is linear but exponentially unstable with control input $u \in \Re^m$ and measurements $y \in \Re^p$. We write the inner plant in state-space form separating the stable modes $(x_s \in \Re^{n_s})$ from the exponentially unstable modes $(x_u \in \Re^{n_u})$ where the dimension of the state vector x is $n = n_s + n_u$,

$$\begin{aligned} \begin{bmatrix} \dot{x}_s \\ \dot{x}_u \end{bmatrix} &= \begin{bmatrix} A_s & A_{su} \\ 0 & A_u \end{bmatrix} \begin{bmatrix} x_s \\ x_u \end{bmatrix} + Bu + Ev + F \\ y &= Cx + Du + Gv. \end{aligned} \tag{1}$$

The vector u of dimension $m = 6$ are the voltage commands for power supplies on the vertical coils, the vector v of dimension $q = 12$ are the voltage demands for the shape coils, the vector y of dimension $p = 7$ consists of the six vertical coil currents and the plasma centroid position. The eigenvalues of A_s have non-positive real part and the eigenvalues of A_u have positive real part.

In addition, we consider that a nominal linear vertical controller has been already designed so that the closed loop system with interconnection conditions $u = y_c$, $u_c = y$ is well posed and internally stable. Due to the composition of the output vector y it is convenient to write the reference for the nominal controller as $r = [r_I \quad r_Z]^T$ where r_I are the current references for the six vertical coils and r_Z is the centroid position reference. When the controller output is subject to saturation, i.e. the interconnection conditions are changed to $u = sat(y_c)$, $u_c = y$, the synthesis of an anti-windup scheme is necessary. In this case the interconnection conditions are modified to

$$u = sat(y_c + v_1), \quad u_c = y + v_2, \tag{2}$$

where the signals v_1 and v_2 are the outputs of the inner-loop anti-windup compensator [20].

Once the components of the outer plant (inner plant + nominal vertical controller + anti-windup compensator) are defined we must focus our attention on its inputs. The shape coil voltage commands v^* coming from the shape controller go through a double saturation stage. The first magnitude saturation is due to the watch-dog that is part of the inner anti-windup augmentation. The second magnitude saturation comes from the hardware limitations of the power supply stage. The vertical coil current references and the centroid position reference r^* imposed by the shape controller also go through a double saturation stage. The first saturation is in magnitude and is due to the physical constraints on the coil currents and centroid position. The second saturation is in rate and is due to the rate limits imposed on the coil current references by the inner anti-windup augmentation. Defining the saturation function

$$\mathrm{sat}_{a^{min}}^{a^{max}}(b) = \begin{cases} a^{max} & \text{if } a^{max} < b \\ b & \text{if } a^{min} \leq b \leq a^{max} \\ a^{min} & \text{if } b < a^{min} \end{cases} \tag{3}$$

we can write the shape coil voltage commands as

$$v = \mathrm{sat}_{M_v^{min}(t)}^{M_v^{max}(t)}(\tilde{v}), \quad \tilde{v} = \mathrm{sat}_{N_v^{min}(t)}^{N_v^{max}(t)}(v^*) \tag{4}$$

where $\tilde{v}$ is the output of the watch-dog block. The saturation levels M_v^{min} and M_v^{max} are determined by the hardware limitations of the power supply stage and are functions of time because they depend on the coil load current and DC supply voltage. The saturation levels N_v^{min} and N_v^{max}, imposed by the watch-dog as it is explained below, are also functions of time because they depend on the value of the unstable mode x_u. The watch-dog is limiting the voltage commands to the shape coils to avoid the loss of controllability of the unstable mode due to the shrinkage of the controllable region. Given the dynamics of the unstable mode

$$\dot{x}_u = A_u x_u + B_u u + E_u v + F_u \tag{5}$$

it is posible to define the controllable region as

$$\overline{\chi} = \left\{ x_u : x_u^{min} \leq x_u \leq x_u^{max} \right\} \tag{6}$$

where

$$x_u^{max} = \frac{-(B_u u)^{max} - E_u v + F_u}{A_u} \tag{7}$$

$$x_u^{min} = \frac{-(B_u u)^{min} - E_u v + F_u}{A_u}. \tag{8}$$

These limiting values are plotted as functions of $E_u v$ in Figure 3. We cannot allow the controllable region to shrink to a null set and we impose a minimum size to it defined as

$$\overline{\chi}_{min} = \left\{ x_u : (x_u^{min})_{max} \leq x_u \leq (x_u^{max})_{min} \right\}. \tag{9}$$

This minimum controllable region defines hard limits on $E_u v$ allowing this signal to be between $(E_u v)_{min}^{lim}$ and $(E_u v)_{max}^{lim}$. In addition we need to impose variable limits on $E_u v$

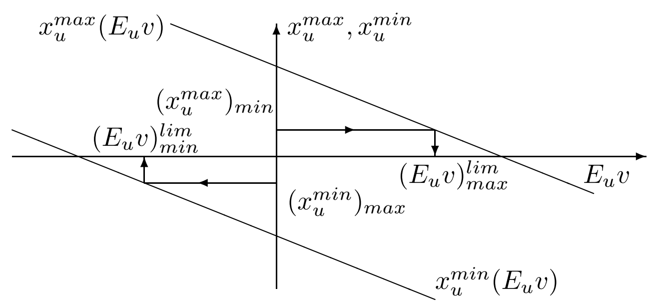


Fig. 3. Controllable region

depending on the position of the unstable mode x_u. Toward this goal we define

$$\begin{aligned} (x_u^{max})^{allowed} &= \max(f_u x_u, (x_u^{max})_{min}) \\ (E_u v)_{max}^{allowed} &= -(B_u u)^{max} + F_u - A_u (x_u^{max})^{allowed} \\ (x_u^{min})^{allowed} &= \min(f_u x_u, (x_u^{min})_{max}) \\ (E_u v)_{min}^{allowed} &= -(B_u u)^{min} + F_u - A_u (x_u^{min})^{allowed} \end{aligned}$$

where $f_u > 1$ is a design constant and for each channel i we compute

$$(N_v^{max})_i = \begin{cases} \frac{(E_u v)_{max}^{allowed}}{\sum_{j=1}^{N_{coils}} |E_{u_j}|} & \text{if } \mathrm{sgn}(E_{u_i}) > 0 \\ -\frac{(E_u v)_{min}^{allowed}}{\sum_{j=1}^{N_{coils}} |E_{u_j}|} & \text{if } \mathrm{sgn}(E_{u_i}) < 0 \end{cases}$$

$$(N_v^{min})_i = \begin{cases} -\frac{(E_u v)_{max}^{allowed}}{\sum_{j=1}^{N_{coils}} |E_{u_j}|} & \text{if } \mathrm{sgn}(E_{u_i}) < 0 \\ \frac{(E_u v)_{min}^{allowed}}{\sum_{j=1}^{N_{coils}} |E_{u_j}|} & \text{if } \mathrm{sgn}(E_{u_i}) > 0. \end{cases}$$

The rate limit on the vertical coil current references aims at preventing the shape controller from asking the system for a response rate that cannot be physically fulfilled. The rate limit for the vertical coil current references can be written as

$$\dot{r}_I = R\,\mathrm{sgn}(\tilde{r}_I - r_I) \tag{10}$$

where R is varied adaptively between maximum and minimum values according to the available control. $\dot{R}$ is equal to $K \min(M_{max} - y_c, y_c - M_{min}) \left|\mathrm{sgn}(\tilde{r}_I - r_I)\right|$ if $M_{min} \leq y_c \leq M_{max}$, 0 if $R > R_{max}$ and $M_{min} \leq y_c \leq M_{max}$, 0 if $R < R_{min}$ and ($y_c < M_{min}$ or $y_c > M_{max}$), $K(M_{max} - y_c)\,|\mathrm{sgn}(\tilde{r}_I - r_I)|$ if $y_c > M_{max}$, and $K(y_c - M_{min})\,|\mathrm{sgn}(\tilde{r}_I - r_I)|$ if $y_c < M_{min}$. $M_{min}(t)$ and $M_{max}(t)$ are the saturation levels for the inner controller, and

$$\tilde{r}_I = \mathrm{sat}_{M_I^{min}}^{M_I^{max}}\left(r_I^*\right). \tag{11}$$

The saturation levels M_I^{min} and M_I^{max} are related to the maximum currents tolerated by the coils. The centroid position reference is also limited by physical constraints due to the finite size of the reactor represented by M_Z^{min} and M_Z^{max}, and is written as

$$r_Z = \mathrm{sat}_{M_Z^{min}}^{M_Z^{max}}\left(r_Z^*\right). \tag{12}$$

IV. Anti-Windup Compensator Fundamentals

We consider the stable plant

$$\begin{aligned} \dot{x} =&\quad f(x,u) \\ y =&\quad h(x,u) \end{aligned} \tag{13}$$

with control input $u \in \Re^m$, measurements $y \in \Re^p$ and states $x \in \Re^n$. In addition, we consider that a nominal controller with state $x_c \in \Re^{n_c}$, input $u_c \in \Re^p$, output $y_c \in \Re^m$ and reference $r \in \Re^p$,

$$\begin{aligned} \dot{x}_c =&\quad g(x_c,u_c,r) \\ y_c =&\quad k(x_c,u_c,r) \end{aligned} \tag{14}$$

has been already designed so that the closed loop system with interconnection conditions

$$u = y_c, \quad u_c = y \tag{15}$$

is well posed and internally stable. The controller performs well locally and succeeds regulating the plant to a desirable value x^* using the control value u^* asymptotically. Both x^* and u^* can be function of time as in the case when an additional known input is driven the system.

Since the plant is stable in this case, the goal of the anti-windup is the modification of the nominal loop with the purpose of avoiding any performance decrease due to the saturation of the actuaction devices. However, it is required that the nominal controller is used and unmodified on a prescribed, not necessarily bounded, neighborhood of (x^*, u^*) denoted by $\mathcal{F}$ where there is no input constraint.

We assume there exist functions F and H and a point x_c^* such that

1) $F(x,u) = f(x,u)$ and $H(x,u) = h(x,u)$ for all $(x,u) \in \mathcal{F}$
2) $y_c^* = k(x_c^*, H(x^*,u^*), r^*)$
3) the feedback interconnection of (14) with the system

$$\begin{aligned} \dot{x} =&\quad F(x,u) \\ y =&\quad H(x,u) \end{aligned} \tag{16}$$

is well-posed and locally Lipschitz and the point (x^*, x_c^*) is globally asymptotically stable.

Being $\mathcal{F}$ the region where there is no input magnitude and/or rate saturation, it is natural to take the modified plant (16) as the input constraint free version of the original plant (13). Since in addition the nominal controller (14) is designed precisely for the input constraint free version of the original plant (modified plant (16)), the three assumptions are satisfied.

The anti-windup augmentation to the controller (14) can be written as

$$\begin{aligned} \dot{x}_{aw}^1 =&\quad f(x_{aw}^1,u) \\ \dot{x}_{aw}^2 =&\quad F(x_{aw}^2,u) \\ s =&\quad -h(x_{aw}^1,u) + H(x_{aw}^2,u) \end{aligned} \tag{17}$$

with initial conditions $x_{aw}^1(0) = x_{aw_o}^1, x_{aw}^2(0) = x_{aw_o}^2$ and with the new interconnection conditions

$$u = y_c, \quad u_c = y + s. \tag{18}$$

Remark 1: Let us consider the system shown in Figure 4 (discard the superscript 's'). We can write the dynamics of our overall plant as

$$\dot{x} \;=\; \begin{bmatrix} \dot{x}_p \\ \dot{x}_a \end{bmatrix} = \begin{bmatrix} f_1(x_p, h_2(x_a,u)) \\ f_2(x_a,u) \end{bmatrix} \equiv f(x,u) \tag{19}$$

$$y \;=\; h_1(x_p,w) = h_1(x_p,h_2(x_a,u)) \equiv h(x,u). \tag{20}$$

As long as $\mathcal{F}$ is contained in or equal to $\Re^n \times \mathcal{U}$, where $\mathcal{U}$ is the set of points u satisfying $u = w$, we may take

$$F(x,u) \;\equiv\; \begin{bmatrix} f_1(x_p,u) \\ f_2(x_a,u) \end{bmatrix} \tag{21}$$

$$H(x,u) \;\equiv\; h_1(x_p,u), \tag{22}$$

which match $f(x,u)$ and $h(x,u)$ on $\Re^n \times \mathcal{U}$. Consequently we can note that if $x_{aw}^1(0) = x(0)$ we will have

$$u_c = y+s = h(x,u) - h(x_{aw}^1,u) + H(x_{aw}^2,u) = H(x_{aw}^2,y_c).$$

The anti-windup, through the signal s, is hiding the saturation in rate and magnitude from the nominal controller and guaranteeing in this way that the controller is well behaved. At its input the controller is seeing the response the plant would have had if no saturation were present.

Remark 2: When $w = u$ we do not want the antiwindup to affect through s the nominal closed loop system. To achieve this goal we must have $s = 0$ and therefore $x_{aw}^1 = x_{aw}^2$. However, in presence of slow modes the system will be affected by the anti-windup for an unnecessarily long time. In this case we modify the anti-windup structure to make $x_e = x_{aw}^1 - x_{aw}^2$, and consequently s, converge to zero arbitrarily fast. Denoting x_{aw}^1 simply as x_{aw}, the anti-windup augmentation can be written now as

$$\begin{aligned} \dot{x}_{aw} =&\quad f(x_{aw},u) \\ \dot{x}_e =&\quad f(x_{aw},u) - F(x_{aw}-x_e,u) + \gamma(u,w)\lambda \\ s =&\quad -h(x_{aw},u) + H(x_{aw}-x_e,u) \\ \lambda =&\quad -cx_e - [f(x_{aw},u) - F(x_{aw}-x_e,u)] \end{aligned} \tag{23}$$

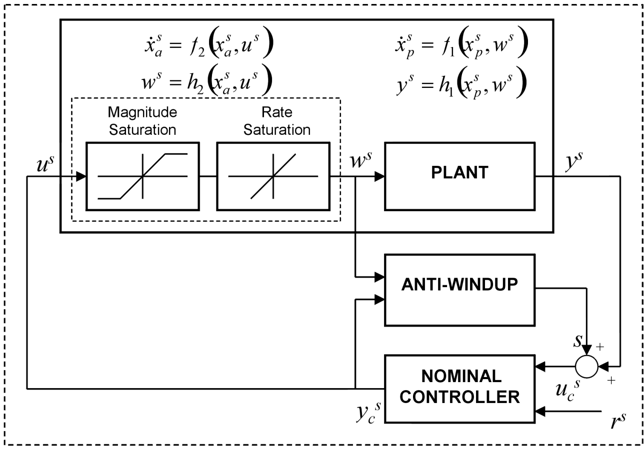


Fig. 4. Anti-windup scheme.

where c is a positive constant and $\gamma(u, w) = 1$ if $u = w$ and 0 otherwise. Recalling that $-h(x_{aw}^1, u) + H(x_{aw}^1, u) = 0$ on $\Re^n \times \mathcal{U}$, i.e. when $w = u$, we guarantee that x_e will converge to zero arbitrarily fast ($\dot{x}_e = -cx_e$) and so will s.

V. Control System Design

Defining the overall input of the outer plant as $w^s = \begin{bmatrix} v^T r_I^T r_Z^T \end{bmatrix}^T$ we can write the dynamics of the outer plant simply as

$$\begin{aligned} \dot{x}_p^s &= f_1(x_p^s, w^s) \\ y^s &= h_1(x_p^s, w^s). \end{aligned}$$

Defining the state and input of the actuators respectively as $x_a^s = r_I$ and $u^s = \begin{bmatrix} (v^*)^T (r_I^*)^T (r_Z^*)^T \end{bmatrix}^T$, we can use equations (4), (10), (11) and (12) to write

$$\begin{aligned} \dot{x}_a^s &= \quad R \,\mathrm{sgn}\left[\mathrm{sat}_{M_I^{min}}^{M_I^{max}}(u_2^s) - x_a\right] & &= f_2(x_a^s, u^s) \\ w^s &= \begin{bmatrix} \mathrm{sat}_{M_v^{min}(t)}^{M_v^{max}(t)}\left[\mathrm{sat}_{N_v^{min}(t)}^{N_v^{max}(t)}(u_1^s)\right] \\ x_a^s \\ \mathrm{sat}_{M_Z^{min}}^{M_Z^{max}}(u_3^s) \end{bmatrix} & &= h_2(x_a^s, u^s). \end{aligned}$$

The superscript "s" has been used with the only purpose of indicating that this is the plant (outer plant) for the shape controller. Examining equation (1) we can note that the dynamics of the inner plant is driven by a constant input. We are in the case then where the equilibrium of the outer plant (x^{s^*}, u^{s^*}) is a function of time $(x^s = [(x_p^s)^T \quad (x_a^s)^T]^T)$.

With the definitions (19-20) and (21-22), the dynamics of the anti-windup augmentation of the shape controller is given (adding the superscript "s") by (23) with interconnection conditions $u^s = y_c^s, \quad u_c^s = y^s + s$. Figure 4 illustrates the architecture of the anti-windup augmentation for the outer loop. The stability of the anti-windup compensator is guaranteed in this case by the stability of the plant.

VI. Conclusions

The necessity of an anti-windup scheme for the outer loop is motivated not only due to the inherent limitations of its actuators but also due to the fact that the inner loop is modifying the control signals of the outer loop in order to preserve stability of the plant. The proposed scheme will be tested in nonlinear simulations first and in experimental conditions later.

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VII. References

[1] J. L. Luxon, "A design retrospective of the DIII-D tokamak", *Nuclear Fusion*, **42**, pp. 614-33, 2002.

[2] D. A. Humphreys, M. L. Walker, J. A. Leuer and J. R. Ferron, "Initial implementation of a multivariable plasma shape and position controller on the DIII-D tokamak", *Proceedings of the 2000 IEEE International Conference on Control Applications*, Anchorage, Alaska, USA, pp. 385-94, September 2000.

[3] R. Albanese and G. Ambrosino, "Current, position and shape control of tokamak plasmas: A literature review", *Proceedings of the 2000 IEEE International Conference on Control Applications*, Anchorage, Alaska, USA, pp. 412-18, September 2000.

[4] L. Scibile and B. Kouvaritakis, "A discrete adaptive near-time optimum control for the plasma vertical position in a tokamak", *IEEE Transactions on control systems technology*, Vol.9, no.1, p. 148, 2001.

[5] M.L. Walker, D.A. Humphreys and E. Schuster, "Some nonlinear Controls for Nonlinear Processes in the DIII-D Tokamak", this conference.

[6] M.L. Walker et al., "Status of DIII-D Plasma Control", *Proceedings of the 16th IEEE/NPSS SOFE*, p. 885, 1995.

[7] G. Ambrosino, M. Ariola, A. Pironti, A. Portone and M. Walker, "A control scheme to deal with coil current saturation in a tokamak", *IEEE Transactions on control systems technology*, Vol.9, no.6, p. 831, November 2001.

[8] R. Albanese, G. Ambrosino, A. Pironti, R. Fressa and A. Portone, "Optimization of the power supply demand for plasma shape control in a tokamak", *IEEE Transactions on magnetics*, Vol.34, no.5, p. 3580, September 1998.

[9] E. G. Gilbert and I. V. Kolmanovsky, "Set-point control of nonlinear systems with state and control constraints: A Lyapunov-function, Reference-Governor approach", *Proceedings of the 38th Conference on Decision and Control*, Phoenix (AZ), USA, pp. 2507-12, Dec. 1999.

[10] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints", *Automatica*, **35**, pp. 407-427, 1999.

[11] L. Scilibe and B. Kouvaritakis, "Stability region for a class of open-loop unstable linear systems: theory and application", *Automatica*, **36**, pp. 37-44, 2000.

[12] J. S. Shamma, "Anti-windup via constrained regulation with observers", *Systems and Control Letters*, **40**, pp. 1869-83, 2000.

[13] A. Zheng, M. V. Kothare and M. Morari, "Anti-windup design for internal model control", *International Journal of Control*, **60**(5), pp. 1015-1024, 1994.

[14] A. Miyamoto and G. Vinnicombe, "Robust control of plants with saturation nonlinearity based on coprime factor representation", *Proceedings of the 36th Conference on Decision and Control*, Kobe, Japan, pp. 2838-40, 1996.

[15] E. F. Mulder, M. V. Kothare and M. Morari, "Multivariable anti-windup controller synthesis using LMI's", *Automatica*, 2001.

[16] A. R. Teel and N. Kapoor, "The $\mathcal{L}_2$ anti-windup problem: Its definition and solution", *Proceedings of the 4th ECC*, Brussels, Belgium, 1997.

[17] A. R. Teel and N. Kapoor, "Uniting local and global controllers", *Proceedings of the 4th ECC*, Brussels, Belgium, 1997.

[18] A. R. Teel, "Anti-Windup for Exponentially Unstable Linear Systems", *International Journal of Robust and Nonlinear Control*, vol.9, pp. 701-716, 1999.

[19] A. R. Teel, "Dynamic anti-windup for nonlinear control systems", *Submitted to IEEE Transaction on Automatic Control*.

[20] E. Schuster, M. L. Walker, D. A. Humphreys and M. Krstic, "Plasma Vertical Stabilization in Presence of Coil Voltage Saturation in the DIII-D Tokamak", *Proceedings of the 2003 American Control Conference*, Denver, Colorado, USA, June 2003.