Nonlinear Burn Control in Tokamaks using In-Vessel Coils

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Abstract— **The tokamak is a magnetic-confinement device where a plasma is confined with the final purpose of generating power from fusion reactions. Unfortunately, working points with favorable fusion conditions in tokamaks are normally found in a region in which the plasma may be thermally unstable. Therefore, regulation of the plasma temperature and density to produce a certain amount of fusion power while avoiding thermal instabilities, known as burn control, is one of the key issues that need to be solved for the success of burning plasma tokamaks such as ITER. Most previous controllers make use of approximate linearization techniques. In the present work, a model-based control approach using nonlinear techniques is proposed. This nonlinear control approach avoids approximate linearization of the model, is applicable to a greater range of operating conditions, and is stable against a larger set of perturbations. In addition to conventional actuation, like modulation of the auxiliary power and modulation of the fueling rate, the in-vessel coil system is considered as a new actuator. The in-vessel coils have the capability to generate non-axisymmetric magnetic fields that modify the plasma confinement, which influences the plasma energy dynamics. A model is proposed to account for the influence that the in-vessel coil actuation has on the plasma confinement. Finally, the effectiveness of the controller is demonstrated via a simulation study for an ITER-like scenario.**

I. Introduction

A tokamak is a particular type of nuclear fusion reactor that makes use of magnetic fields to confine a reactant gas in a torus-shaped vessel, so that fusion reactions occur frequently enough [1]. The reactants, typically deuterium (D) and tritium (T), must be heated to extremely high temperatures ($\sim$ 10 million degrees) by means of auxiliary sources, so they overcome Coulombic repulsion forces and fuse. In that process, Helium ions (α-particles) are produced, as well as neutrons and energy. In order to have a commercially viable tokamak power plant, operation at working points with high fusion gain Q is required, where Q is the ratio of fusion power to auxiliary power. The definition of these working points by means of a desired plasma density and temperature determines a target burn condition around which the system needs to be regulated. ITER, the next step in nuclear fusion research, may require operation at working points characterized by low temperature and high density where the burning plasma may be thermally unstable. While operating at these working points, the fusion reactor can suffer certain thermal runaway conditions, like excursions (positive runaways in temperature) or quenches (negative runaways in temperature). In the case of an excursion, too high temperatures are reached, which may lead to plasma disruptions. In the case of a quench, low fusion power and poor performance are found, potentially leading to a complete shutoff of the reactor. Also, plasma disruptions may be triggered during a quench, causing damage to the confinement vessel walls [1]. Therefore, control of the plasma density and temperature, usually referred to as burn control, is critical not only to regulate the desired amount of fusion power, but also to prevent the fusion reactor from potentially suffering thermal instabilities.

Past work considered different combinations of the available actuators for the regulation of the burn condition. Each actuation method has its own advantages and drawbacks. For example, in [2] only modulation of the auxiliary power was used. This approach is appropriate as long as the desired working point is not characterized by a too small value of the auxiliary power; if that is the case, the plasma energy cannot be decreased by means of modulation of the auxiliary power, and a thermal excursion cannot be prevented if no other actuator is used. Other work, like [3], proposed a control approach based uniquely on the modulation of the D-T fueling rate. This method is suitable provided that disruptive density limits are not reached. An additional actuation option is the controlled injection of impurities, which can be used to increase the radiative losses, cooling the plasma and preventing thermal excursions. Comprehensive control strategies using these three actuators were considered in [4].

In most previous pieces of work, the model was approximately linearized to make use of linear techniques, which limits the size of the perturbations that can be stabilized. A nonlinear control design can guarantee a substantially larger region of attraction than a linear control design. In our previous works [5], [6], two different nonlinear controllers using all the previously mentioned actuators were synthesized. In [5], the model always considered a 50:50 D-T mix. Moreover, the recycling effects caused by particles coming from the plasma facing components of the confinement vessel walls were not considered. In [6], both variations in the D-T mixture and recycling effects were taken into account in the model, and a controller based on isotopic fuel tailoring was proposed. However, the combined use of impurities injection and isotopic fueling may still be required under certain conditions to reject thermal excursions and stabilize the plasma energy.

Injecting impurities has important drawbacks. Large perturbations in the plasma energy normally require the injection of a large amount of impurities in order to avoid a thermal ex-

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cursion. These particles remain in the plasma for a long time, increasing the needed auxiliary power to compensate for the radiative losses produced by the impurities and reducing Q. These drawbacks require that alternative control methods be studied. Based on some encouraging experimental results [7], the use of the in-vessel coils comes up as an alternative to reduce the plasma energy. The in-vessel coils, currently available in present-day devices such as DIII-D [8] and expected in ITER [9], can reduce the confinement time of the particles by generating non-axisymmetric magnetic fields in the plasma. This reduced confinement time of the particles implies an effective decrease in the energy confinement time, resulting in a faster decrease of the plasma energy. So far, most experiments and studies have used the in-vessel coils for purposes different from burn control, such as the control of instabilities like resistive wall modes or edge localized modes [10], [11], or plasma position and shape control [12]. In this work, a nonlinear feedback controller for the regulation of the burn condition is proposed. The main novelty of this controller is the inclusion of the in-vessel coil system as an actuator, together with modulation of the auxiliary power and modulation of the fueling rate.

The paper is organized as follows. The burning plasma model is described in Section II. In Section III, the control objective and the proposed control algorithm are stated. The effectiveness of the controller is demonstrated in Section IV via simulations. Finally, some conclusions and possible plans for future work are presented in Section V.

II. Model description

The model described in this work expands prior modeling work [6] to include the effects of in-vessel coil actuation. All the magnitudes referred to are volume-averaged magnitudes.

The balance equation for the α-particle density, n_α, is

$$\dot{n}_\alpha = -\frac{n_\alpha}{\tau_\alpha^*} + S_\alpha, \tag{1}$$

where τ_α^* is the α-particles confinement time, and S_α is the source of α-particles from fusion, which is given by

$$S_\alpha = \gamma\,(1-\gamma)\,n_{DT}^2\langle\sigma v\rangle, \tag{2}$$

where n_{DT} is the total deuterium-tritium density ($n_{\mathrm{DT}} = n_{\mathrm{D}} + n_{\mathrm{T}}$, where n_{D} is the deuterium density and n_{T} is the tritium density), γ is the tritium fraction ($\gamma = n_{\mathrm{T}}/n_{\mathrm{DT}}$), and $\langle\sigma\nu\rangle$ is the DT reactivity, which is a function of the plasma temperature, T, and is computed by

$$\langle\sigma\nu\rangle = \exp\left(\frac{a_1}{T^r} + a_2 + a_3T + a_4T^2 + a_5T^3 + a_6T^4\right), \tag{3}$$

where a_i and r are constant scaling parameters [13].

The deuterium and tritium particle densities, n_{D} and n_{T}, are treated separately, and their balance equations are

$$\dot{n}_{\mathrm{D}} = -\frac{n_{\mathrm{D}}}{\tau_{\mathrm{D}}} + f_{\mathrm{eff}}S_{\mathrm{D}}^{\mathrm{R}} - S_\alpha + S_{\mathrm{D}}^{\mathrm{inj}}, \tag{4}$$

$$\dot{n}_{\mathrm{T}} = -\frac{n_{\mathrm{T}}}{\tau_{\mathrm{T}}} + f_{\mathrm{eff}}S_{\mathrm{T}}^{\mathrm{R}} - S_\alpha + S_{\mathrm{T}}^{\mathrm{inj}}, \tag{5}$$

where τ_{D} and τ_{T} are, respectively, the deuterium and tritium particles confinement times, $S_{\mathrm{D}}^{\mathrm{inj}}$ and $S_{\mathrm{T}}^{\mathrm{inj}}$ are, respectively, the deuterium and tritium particles injection rates, $S_{\mathrm{D}}^{\mathrm{R}}$ and $S_{\mathrm{T}}^{\mathrm{R}}$ are the deuterium and tritium particles sources from recycling, respectively, and f_{eff} is a parameter that represents the efficiency with which the recycled particles fuel the plasma core. It is considered that $S_{\mathrm{D}}^{\mathrm{inj}}$ and $S_{\mathrm{T}}^{\mathrm{inj}}$ can be directly controlled and are inputs to the model. The recycling sources for D and T are modeled as

$$S_{\mathrm{D}}^{\mathrm{R}} = \frac{1}{1-f_{\mathrm{ref}}(1-f_{\mathrm{eff}})}\Bigg\{f_{\mathrm{ref}}\frac{n_{\mathrm{D}}}{\tau_{\mathrm{D}}} + (1-\gamma_{\mathrm{PFC}}) \left[\frac{(1-f_{\mathrm{ref}}(1-f_{\mathrm{eff}}))R_{\mathrm{eff}}}{1-R_{\mathrm{eff}}(1-f_{\mathrm{eff}})} - f_{\mathrm{ref}}\right]\left(\frac{n_{\mathrm{D}}}{\tau_{\mathrm{D}}} + \frac{n_{\mathrm{T}}}{\tau_{\mathrm{T}}}\right)\Bigg\}, \tag{6}$$

$$S_{\mathrm{T}}^{\mathrm{R}} = \frac{1}{1-f_{\mathrm{ref}}(1-f_{\mathrm{eff}})}\Bigg\{f_{\mathrm{ref}}\frac{n_{\mathrm{T}}}{\tau_{\mathrm{T}}} + \gamma_{\mathrm{PFC}} \left[\frac{(1-f_{\mathrm{ref}}(1-f_{\mathrm{eff}}))R_{\mathrm{eff}}}{1-R_{\mathrm{eff}}(1-f_{\mathrm{eff}})} - f_{\mathrm{ref}}\right]\left(\frac{n_{\mathrm{D}}}{\tau_{\mathrm{D}}} + \frac{n_{\mathrm{T}}}{\tau_{\mathrm{T}}}\right)\Bigg\}, \tag{7}$$

where f_{ref}, R_{eff} and γ_{PFC} are recycling parameters that characterize the recycling effects [6].

The balance for the plasma energy, E, is given by

$$\dot{E} = -\frac{E}{\tau_E} + P_\alpha + P_{\mathrm{Ohm}} - P_{\mathrm{rad}} + P_{\mathrm{aux}} = -\frac{E}{\tau_E} + P, \tag{8}$$

where τ_E is the energy confinement time, P_α is the contribution from α-particle heating, P_{Ohm} is the ohmic heating power, P_{rad} is the radiative power loss, P_{aux} is the auxiliary heating power and $P = P_\alpha + P_{\mathrm{Ohm}} - P_{\mathrm{rad}} + P_{\mathrm{aux}}$ is the total power. The expression for the α-particle power is $P_\alpha = Q_\alpha\gamma\,(1-\gamma)\,n_{\mathrm{DT}}^2\langle\sigma\nu\rangle = Q_\alpha S_\alpha$, where $Q_\alpha = 3.52$ MeV is the energy of the α-particles produced by the fusion reaction. The ohmic heating power is approximated as $P_{\mathrm{Ohm}} = 2.8\times10^{-9}\frac{Z_{\mathrm{eff}}I_p^2}{a^4T^{3/2}}$, where I_p is the plasma current, a is the minor radius of the tokamak, T has to be given in keV and Z_{eff}, the effective atomic number of the plasma ions, is given by

$$Z_{\mathrm{eff}} = \left(n_{\mathrm{D}} + n_{\mathrm{T}} + 4n_\alpha + n_{\mathrm{I}}Z_{\mathrm{I}}^2\right)/n_{\mathrm{e}}. \tag{9}$$

On the right hand side of (9), n_{I} is the total impurity density, and n_{e} is the electron density, that can be obtained from the neutrality condition as

$$n_{\mathrm{e}} = n_{\mathrm{D}} + n_{\mathrm{T}} + 2n_\alpha + Z_{\mathrm{I}}n_{\mathrm{I}}, \tag{10}$$

where Z_{I} is the atomic number of the impurities. For the radiative loss, the approximation given in [14] is used, where the expression for P_{rad} is taken as the combination of bremsstrahlung, line and recombination losses, $P_{\mathrm{rad}} = P_{\mathrm{brem}} + P_{\mathrm{line}} + P_{\mathrm{rec}}$, and each component is given by

$$P_{\mathrm{brem}} = 4.8\times10^{-37}\Big(\sum_i n_iZ_i^2\Big)n_{\mathrm{e}}\sqrt{T}, \tag{11}$$

$$P_{\mathrm{line}} = 1.8\times10^{-38}\Big(\sum_i n_iZ_i^4\Big)n_{\mathrm{e}}\,T^{-1/2}, \tag{12}$$

$$P_{\mathrm{rec}} = 4.1\times10^{-40}\Big(\sum_i n_iZ_i^6\Big)n_{\mathrm{e}}\,T^{-3/2}, \tag{13}$$

where T has to be given in keV.

The balance equation for the total impurity density, n_I, is expressed as

$$\dot{n}_\mathrm{I} \quad = \quad -\frac{n_\mathrm{I}}{\tau_\mathrm{I}^*} + S_\mathrm{I}^\mathrm{inj} + S_\mathrm{I}^\mathrm{sp}. \qquad (14)$$

where τ_I^* is the impurity particles confinement time, $S_\mathrm{I}^\mathrm{inj}$ is the controllable rate of injection of impurities and S_I^sp is the source of impurities arising from confinement vessel walls sputtering that cannot be directly controlled, and is given by

$$S_\mathrm{I}^\mathrm{sp} = \frac{f_\mathrm{I}^\mathrm{sp} n}{\tau_\mathrm{I}^*} + f_\mathrm{I}^\mathrm{sp}\dot{n}, \qquad (15)$$

where n is the total plasma density,

$$n = n_\mathrm{i} + n_\mathrm{e} = 2n_\mathrm{D} + 2n_\mathrm{T} + 3n_\alpha + (Z_\mathrm{I}+1)n_\mathrm{I}, \qquad (16)$$

where $n_\mathrm{i} = n_\mathrm{D} + n_\mathrm{T} + n_\alpha + n_\mathrm{I}$ is the ions total density, and f_I^sp is a parameter that represents the fact that there is always some content of impurities in the plasma.

The expression that relates T with n and E is

$$E = \frac{3}{2}nT. \qquad (17)$$

For τ_E, many scalings have been proposed, like in [15]. Most of them have a shape given by $\tau_E = H_H \times K \times P^{-N}$, where

- H_H is a constant that represents differences in the possible experimental conditions that are not included in the rest of the parameters of the scaling. It is normally a measure of the plasma confinement quality.
- K is a nonlinear function of the machine parameters, which are considered to be known and constant in this model, and also of other variables such as γ and/or n_e. The shape of this function depends on the scaling used.
- N is a constant that depends on the scaling used.

In this work, the effect of the in-vessel coil actuation on τ_E is modeled by modifying H_H. The modified energy confinement time is denoted as τ_E^*, and is given by

$$\tau_E^* = H_H^*(I_\mathrm{coils}) \times K \times P^{-N}, \qquad (18)$$

where I_coils is the electric current driven through the in-vessel coils, and the function $H_H^*(I_\mathrm{coils})$ is determined from experimental data. A suitable expression is

$$H_H^*(I_\mathrm{coils}) = \mathrm{A}I_\mathrm{coils}^2 + \mathrm{B}I_\mathrm{coils} + \mathrm{C}, \qquad (19)$$

where A, B and C are experiment-based [7] coefficients that must be determined to best fit the behavior of τ_E^* when the in-vessel coils are operated. It is considered that τ_E^*, and then $H_H^*(I_\mathrm{coils})$, are maximum at a desired working point when the in-vessel coils are not operated, i.e., the in-vessel coils cannot improve the plasma confinement at the desired working point. Then, when $I_\mathrm{coils} = 0$, it is found that $H_H^*(0) = H_H = C$. Also, it is considered that there is a minimum value achievable for $H_H^*(I_\mathrm{coils})$ which corresponds to the maximum current that can be driven through the in-vessel coils, $I_\mathrm{coils}^\mathrm{max}$.

Using the notation introduced, (8) becomes

$$\dot{E} = -\frac{E}{\tau_E^*} + P_\alpha + P_\mathrm{Ohm} - P_\mathrm{rad} + P_\mathrm{aux}. \qquad (20)$$

Finally, it is assumed that all particle confinement times scale with τ_E^* as

$$\tau_\alpha^* = k_\alpha^*\tau_E^*\ ,\ \tau_D = k_D\tau_E^*\ ,\ \tau_T = k_T\tau_E^*\ ,\ \tau_I^* = k_I^*\tau_E^*, \quad (21)$$

where k_α^*, k_D, k_T and k_I^* are scaling constants, and the superscript (*) indicates that the scaling constant takes into account the related particle recycling effects. Therefore, it is considered that τ_α^* and τ_I^* include the recycling effects for these type of particles, while τ_D and τ_T do not include those effects, which are explicitly modeled using (6) and (7). Note that, for τ_E^*, the superscript (*) is not related to the recycling effects, but to the in-vessel coil actuation.

III. Control Algorithm

A. Control Objective

The primary control goal is to drive the burning plasma to a desired working point (starting from either another working point or a certain set of perturbed initial conditions), and maintain the system at that working point. The achievable working points of the reactor are characterized by the equilibrium of the model equations (1), (4), (5), (14) and (20),

$$\begin{aligned}
0 &= -\frac{\bar{n}_\alpha}{\bar{\tau}_\alpha^*} + \bar{S}_\alpha, \\
0 &= -\frac{\bar{n}_\mathrm{D}}{\bar{\tau}_\mathrm{D}} + f_\mathrm{eff}\bar{S}_\mathrm{D}^\mathrm{R} - \bar{S}_\alpha + \bar{S}_\mathrm{D}^\mathrm{inj}, \\
0 &= -\frac{\bar{n}_\mathrm{T}}{\bar{\tau}_\mathrm{T}} + f_\mathrm{eff}\bar{S}_\mathrm{T}^\mathrm{R} - \bar{S}_\alpha + \bar{S}_\mathrm{T}^\mathrm{inj}, \\
0 &= -\frac{\bar{n}_\mathrm{I}}{\bar{\tau}_\mathrm{I}^*} + \bar{S}_\mathrm{I}^\mathrm{inj} + \bar{S}_\mathrm{I}^\mathrm{sp}, \\
0 &= -\frac{\bar{E}}{\bar{\tau}_E^*} + \bar{P}_\alpha + \bar{P}_\mathrm{Ohm} - \bar{P}_\mathrm{rad} + \bar{P}_\mathrm{aux},
\end{aligned} \qquad (22)$$

where the bar in all variables indicates equilibrium values. The set of equations (22) that characterizes the equilibrium consists of five equations and nine unknowns. In the normal operation of a fusion reactor, it is desired that $\bar{S}_\mathrm{I}^\mathrm{inj} = 0$. Hence, it is necessary to specify three more variables to solve for the equilibrium.

By introducing $n_\alpha = \bar{n}_\alpha + \tilde{n}_\alpha$, $n_\mathrm{D} = \bar{n}_\mathrm{D} + \tilde{n}_\mathrm{D}$, $n_\mathrm{T} = \bar{n}_\mathrm{T} + \tilde{n}_\mathrm{T}$, $n_\mathrm{I} = \bar{n}_\mathrm{I} + \tilde{n}_\mathrm{I}$ and $E = \bar{E} + \tilde{E}$, where the tilde in all variables indicates deviation values with respect to the equilibrium values, the dynamic equations (1), (4), (5), (14) and (20) are rewritten as

$$\begin{aligned}
\dot{\tilde{n}}_\alpha &= -\frac{\bar{n}_\alpha}{\tau_\alpha^*} - \frac{\tilde{n}_\alpha}{\tau_\alpha^*} + S_\alpha, \\
\dot{\tilde{n}}_\mathrm{D} &= -\frac{\bar{n}_\mathrm{D}}{\tau_\mathrm{D}} - \frac{\tilde{n}_\mathrm{D}}{\tau_\mathrm{D}} + f_\mathrm{eff}S_\mathrm{D}^\mathrm{R} - S_\alpha + S_\mathrm{D}^\mathrm{inj}, \\
\dot{\tilde{n}}_\mathrm{T} &= -\frac{\bar{n}_\mathrm{T}}{\tau_\mathrm{T}} - \frac{\tilde{n}_\mathrm{T}}{\tau_\mathrm{T}} + f_\mathrm{eff}S_\mathrm{T}^\mathrm{R} - S_\alpha + S_\mathrm{T}^\mathrm{inj}, \\
\dot{\tilde{n}}_\mathrm{I} &= -\frac{\bar{n}_\mathrm{I}}{\tau_\mathrm{I}^*} - \frac{\tilde{n}_\mathrm{I}}{\tau_\mathrm{I}^*} + S_\mathrm{I}^\mathrm{inj} + S_\mathrm{I}^\mathrm{sp}, \\
\dot{\tilde{E}} &= -\frac{\bar{E}}{\tau_E^*} - \frac{\tilde{E}}{\tau_E^*} + P_\alpha + P_\mathrm{Ohm} - P_\mathrm{rad} + P_\mathrm{aux}.
\end{aligned} \qquad (23)$$

Driving this autonomous, nonlinear system (23) to zero equates to driving the original system (1), (4), (5), (14) and (20) to the desired equilibrium.

B. Controller Design

The control strategy proposed in the present work integrates the following three actuation methods: modulation of the auxiliary power modulation, actuation of the in-vessel coils and modulation of the D and T fueling rates.

First, the controller attempts to regulate the plasma energy deviation by modulating the auxiliary power. The equation for $\tilde{E}$ in system (23) is reduced to

$$\dot{\tilde{E}} = -(\frac{1}{\tau_E^*} + K_E)\tilde{E}, \tag{24}$$

just by setting

$$-\frac{\bar{E}}{\tau_E^*} + P = -K_E\tilde{E}, \tag{25}$$

where $K_E > 0$ is a design parameter. As $\tau_E^* > 0$, equation (24) yields that the energy subsystem is exponentially stable as long as condition (25) is fulfilled. This is achieved with the control law

$$P_{\text{aux}}^{\text{unsat}} = -K_E\tilde{E} - \frac{\bar{E}}{\tau_E^*} - P_\alpha - P_{\text{Ohm}} + P_{\text{rad}}. \tag{26}$$

Then, by setting $P_{\text{aux}} = P_{\text{aux}}^{\text{unsat}}$, the stability of the energy subsystem is assured. However, there exist physical saturation limits for P_{aux}, and it may not be possible to set $P_{\text{aux}} = P_{\text{aux}}^{\text{unsat}}$. The maximum and minimum achievable values for P_{aux} are denoted by $P_{\text{aux}}^{\text{max}}$ and $P_{\text{aux}}^{\text{min}}$, respectively. If $P_{\text{aux}}^{\text{unsat}} > P_{\text{aux}}^{\text{max}}$, the controller keeps $P_{\text{aux}} = P_{\text{aux}}^{\text{max}}$, but the energy subsystem stability cannot be guaranteed. If the energy subsystem is not stabilized after some time even with $P_{\text{aux}} = P_{\text{aux}}^{\text{max}}$, it is likely that more auxiliary power needs to be installed in the machine to reach such working point, or that the machine parameters need to be improved. If, on the contrary, $P_{\text{aux}}^{\text{unsat}} < P_{\text{aux}}^{\text{min}}$, the controller keeps $P_{\text{aux}} = P_{\text{aux}}^{\text{min}}$, and the energy subsystem stability cannot be guaranteed only by means of modulation of the auxiliary power. In that case, the controller makes use of the in-vessel coils. The equation for $\tilde{E}$ in system (23) is reduced to

$$\dot{\tilde{E}} = -(\frac{1}{\tau_E^*} + K_{\tau_E})\tilde{E}, \tag{27}$$

just by setting

$$-\frac{\bar{E}}{\tau_E^*} + P^{\text{min}} = -K_{\tau_E}\tilde{E}, \tag{28}$$

where $K_{\tau_E} > 0$ is a design parameter and the superscript $(\cdot)^{\text{min}}$ denotes that P is computed with $P_{\text{aux}} = P_{\text{aux}}^{\text{min}}$. As $\tau_E^* > 0$, equation (27) yields that the energy subsystem is exponentially stable as long as condition (28) is fulfilled, which can be achieved by setting the energy confinement time τ_E^* to

$$\tau_E^* = \frac{\bar{E}}{P^{\text{min}} + K_{\tau_E}\tilde{E}}, \tag{29}$$

which, in conjunction with (18) and (19), allow for the computation of $I_{\text{coils}}^{\text{unsat}}$, which is the value of I_{coils} that stabilizes the energy subsystem. If the in-vessel coils saturate, that is, if $I_{\text{coils}}^{\text{unsat}} > I_{\text{coils}}^{\text{max}}$, the controller cannot guarantee the stability of the energy subsystem. In order to further reduce the plasma energy, alternative techniques such as isotopic fueling or impurities injection could be considered. Then, barring those situations that require too large injections of auxiliary power or too large decreases of the plasma energy and, therefore, there is not enough actuation capability, it can be assured that the plasma energy subsystem is stabilized just by modulation of the auxiliary power and in-vessel coil actuation.

To stabilize both D and T density subsystems, the controller uses modulation of the fueling rate. By choosing $S_{\text{D}}^{\text{inj}}$ and $S_{\text{T}}^{\text{inj}}$ as

$$S_{\text{D}}^{\text{inj,unsat}} = \frac{\bar{n}_{\text{D}}}{\tau_{\text{D}}} - f_{\text{eff}}S_{\text{D}}^{\text{R}} + S_\alpha - K_{\text{D}}\tilde{n}_{\text{D}}, \tag{30}$$

$$S_{\text{T}}^{\text{inj,unsat}} = \frac{\bar{n}_{\text{T}}}{\tau_{\text{T}}} - f_{\text{eff}}S_{\text{T}}^{\text{R}} + S_\alpha - K_{\text{T}}\tilde{n}_{\text{T}}, \tag{31}$$

where $K_{\text{D}}, K_{\text{T}} > 0$ are design parameters, the $\tilde{n}_{\text{D}}$ and $\tilde{n}_{\text{T}}$ equations in (23) are reduced to

$$\dot{\tilde{n}}_{\text{D}} = -\frac{\tilde{n}_{\text{D}}}{\tau_{\text{D}}} - K_{\text{D}}\tilde{n}_{\text{D}} = -\left(\frac{1}{\tau_{\text{D}}} + K_{\text{D}}\right)\tilde{n}_{\text{D}}, \tag{32}$$

$$\dot{\tilde{n}}_{\text{T}} = -\frac{\tilde{n}_{\text{T}}}{\tau_{\text{T}}} - K_{\text{T}}\tilde{n}_{\text{T}} = -\left(\frac{1}{\tau_{\text{T}}} + K_{\text{T}}\right)\tilde{n}_{\text{T}}, \tag{33}$$

achieving exponential stability for both D and T density subsystems ($\tau_{\text{D}}, \tau_{\text{T}} > 0$). However, physical saturation limits also exist which may make impossible to set $S_{\text{D}}^{\text{inj}} = S_{\text{D}}^{\text{inj,unsat}}$ and $S_{\text{T}}^{\text{inj}} = S_{\text{T}}^{\text{inj,unsat}}$. Barring those situations in which the D and T fueling injectors are saturated for too long periods of time, it can be assured that the D and T density subsystems are stabilized just by modulation of the fueling rate.

To finally close the controller design, it is shown that $n_\alpha \to \bar{n}_\alpha$ and $n_{\text{I}} \to \bar{n}_{\text{I}}$ in time, provided that stability of the energy and D and T subsystems is achieved, i.e., $n_{\text{D}} \to \bar{n}_{\text{D}}$, $n_{\text{T}} \to \bar{n}_{\text{T}}$ and $E \to \bar{E}$. First, by defining $\hat{n}_{\text{I}} = n_{\text{I}} - f_{\text{I}}^{\text{sp}}n$, and taking into account that $S_{\text{I}}^{\text{inj}} = 0$, (14) can be rewritten as

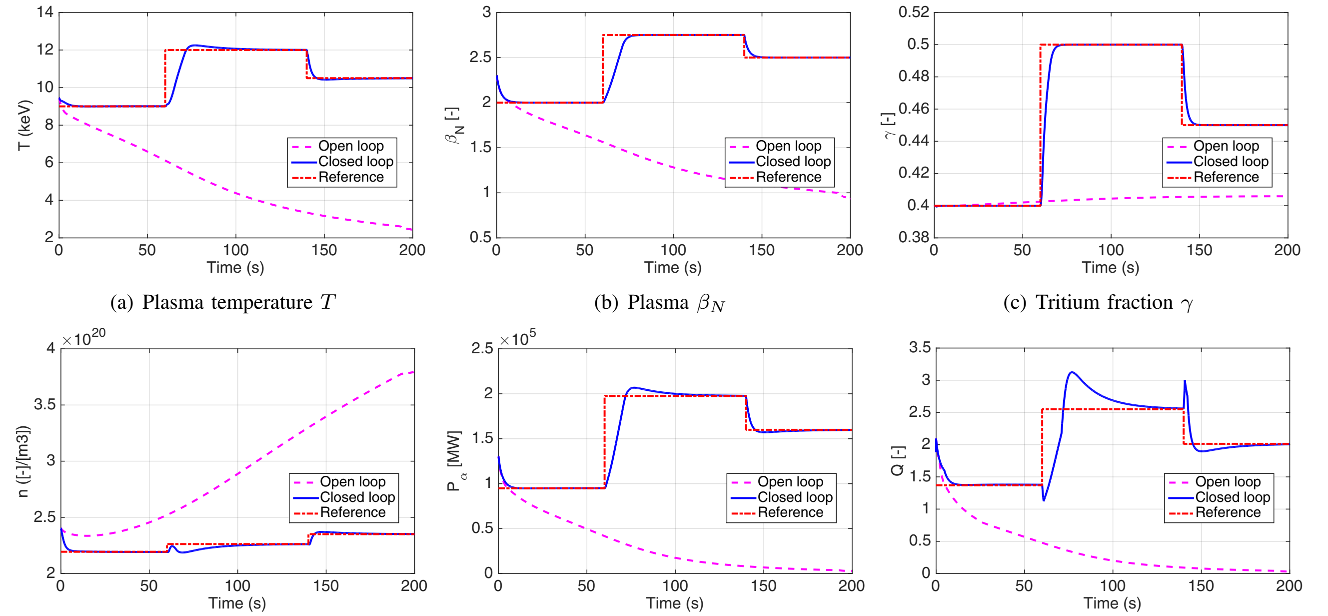
$$\dot{\hat{n}}_{\text{I}} + f_{\text{I}}^{\text{sp}}\dot{n} = -\frac{\hat{n}_{\text{I}} + f_{\text{I}}^{\text{sp}}n}{\tau_{\text{I}}^*} + S_{\text{I}}^{\text{sp}}, \tag{34}$$

and using (15), it is found that $\dot{\hat{n}}_I = -\frac{\hat{n}_I}{\tau_I^*}$, i.e., $\hat{n}_{\text{I}} \to 0$ exponentially fast ($\tau_{\text{I}}^* > 0$), and then, $n_{\text{I}} \to f_{\text{I}}^{\text{sp}}n$ exponentially fast. Second, by focusing on (1), it can be noted that positive perturbations in n_α ($\tilde{n}_\alpha > 0$) decrease the first term $-\frac{n_\alpha}{\tau_\alpha^*}$. For the second term S_α, it can be noticed that as $n_{\text{D}} \to \bar{n}_{\text{D}}$ and $n_{\text{T}} \to \bar{n}_{\text{T}}$, then $n_{\text{DT}} \to \bar{n}_{\text{DT}}$ and $\gamma \to \bar{\gamma}$. Thus, from (2), $S_\alpha \to \bar{\gamma}(1-\bar{\gamma})\bar{n}_{\text{DT}}\langle\sigma\nu\rangle$. For the range of interest, $\langle\sigma\nu\rangle$ is an increasing function of T, equation (3). Taking into account that $n_{\text{I}} \to f_{\text{I}}^{\text{sp}}n$, (10) and (16) yield

$$\lim_{n_{\text{I}} \to f_{\text{I}}^{\text{sp}}n} n = \frac{3(\bar{n}_\alpha + \tilde{n}_\alpha) + 2\bar{n}_{\text{DT}}}{1 - f_{\text{I}}^{\text{sp}}(1 + Z_{\text{I}})}. \tag{35}$$

It can be noted that $\tilde{n}_\alpha > 0$ will imply an increase in n. Thus, using $E \to \bar{E}$, equation (17) becomes

$$T = \frac{\bar{E}}{\frac{3}{2}n}, \tag{36}$$



(a) Plasma temperature T (b) Plasma β_N (c) Tritium fraction γ

(d) Plasma density n (e) α-heating power P_α (f) Fusion gain Q

Fig. 1. Time evolution for (a) plasma temperature T, (b) β_{N}, (c) tritium fraction γ, (d) plasma density n, (e) α-heating power P_α, and (f) fusion gain Q.

and it can be concluded that T decreases, and also that $\langle\sigma\nu\rangle$ decreases. Then, S_α decreases too. On the other hand, for negative perturbations in n_α ($\tilde{n}_\alpha < 0$), $-\frac{n_\alpha}{\tau_\alpha^*}$ increases, and n decreases, T increases, $\langle\sigma\nu\rangle$ increases and S_α increases. This allows to write equation (1) as $\dot{\tilde{n}}_\alpha = -\phi_\alpha \tilde{n}_\alpha$, where ϕ_α is some positive function. As $\phi_\alpha > 0$, the α-particles density subsystem is exponentially stable. Therefore, it can be concluded that $\tilde{n}_\alpha \to 0$ and $n_\alpha \to \bar{n}_\alpha$. To finish this stability proof, from equation (35), it can be seen that, $n \to \frac{3\bar{n}_\alpha + 2\bar{n}_{\mathrm{DT}}}{1 - f_{\mathrm{I}}^{\mathrm{sp}}(1+Z_{\mathrm{I}})} = \bar{n}$, and finally, that $n_{\mathrm{I}} \to f_{\mathrm{I}}^{\mathrm{sp}} \bar{n} = \bar{n}_{\mathrm{I}}$.

IV. Simulation Results

A simulation study is carried out to show the performance and disturbance rejection capability of the controller. The machine parameters are taken as I_p = 15.0 MA, a = 2.0 m, R = 6.2 m (major radius), B_{T} = 5.3 T (toroidal magnetic field), κ_{95} = 1.7 (elongation at the 95% flux surface/separatrix), and V = 837 m^3 (plasma volume). The energy confinement time scaling ($k_{\tau_E} = 0.082$) used is

$$\tau_E^* = k_{\tau_E} H_H^* \times I_p^{1.02} B_{\mathrm{T}}^{0.15} M^{0.5} R^{1.6} \kappa_{95}^{-0.19} \times (PV)^{-0.47}, \quad (37)$$

where M is the effective mass of the plasma in amu [15]. The recycling parameters are $f_{\mathrm{I}}^{\mathrm{sp}}$ = 0.02, R_{eff} = 0.9, γ_{PFC} = 0.4, f_{ref} = 0.7 and f_{eff} = 0.3, and k_α^* = 7, k_{D} = 3, k_{T} = 3, k_{I}^* = 10 and Z_{I} = 4. The actuator limits are shown in Table I. It is assumed that the in-vessel coils have the same capability to modify H_H^* as in the experiments carried out in [7]. Based on such assumption, and using $H_H = 0.75$, which is a reasonable value for this scenario, the expression for $H_H^*(I_{\mathrm{coils}})$ is found to be

$$H_H^*(I_{\mathrm{coils}}) = 0.010313 I_{\mathrm{coils}}^2 - 0.099375 I_{\mathrm{coils}} + 0.75. \quad (38)$$

Also, it is convenient to introduce the plasma $\beta_{\mathrm{N}} = 1.33\mu_0 a E/(I_p B_{\mathrm{T}})$ as a magnitude to define the plasma equilibrium, where μ_0 is the vacuum magnetic permeability.

In this simulation study, the system starts from a perturbed initial condition and is first driven to a working point characterized by $\bar{T}$ = 9 keV, $\bar{\beta}_{\mathrm{N}}$ = 2 and $\bar{\gamma}$ = 0.4. The initial conditions are -20% in $\bar{n}_\alpha$, +10% in $\bar{n}_{\mathrm{D}}$ and $\bar{n}_{\mathrm{T}}$, and +15% in $\bar{E}$. No initial perturbation in n_{I} is considered. At t = 60 sec., the system is driven to a second working point characterized by $\bar{T}$ = 12 keV, $\bar{\beta}_{\mathrm{N}}$ = 2.75 and $\bar{\gamma}$ = 0.5. At t = 140 sec., the system is driven to a third working point defined by $\bar{T}$ = 10.5 keV , $\bar{\beta}_{\mathrm{N}}$ = 2.5 and $\bar{\gamma}$ = 0.45. Fig. 1 shows evolutions for T, β_{N}, γ, n, P_α and Q in open loop and closed loop simulations, together with the reference to the system. The required inputs $S_{\mathrm{D}}^{\mathrm{inj}}$, $S_{\mathrm{T}}^{\mathrm{inj}}$, I_{coils} and P_{aux} are shown in Fig. 2. Note that, in spite of the saturation of $S_{\mathrm{D}}^{\mathrm{inj}}$, $S_{\mathrm{T}}^{\mathrm{inj}}$ and P_{aux} for short periods of time, the controller successfully stabilizes and drives the system between working points, rejecting the perturbation in the initial conditions. Also note that, as suggested in [7], really small values of I_{coils} seem to be necessary to control the plasma energy, and the activation of the in-vessel coils is required during small time windows.

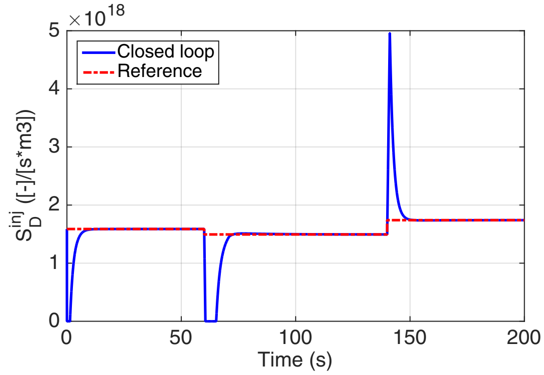
TABLE I

Actuator Limits

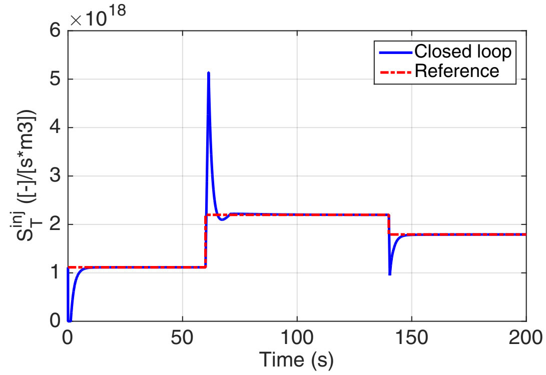
Variable	Value
$P_{\mathrm{aux}}^{\mathrm{max}}$	73 MW
$P_{\mathrm{aux}}^{\mathrm{min}}$	$5/7 \times P_{\mathrm{aux}}^{\mathrm{min}}$
$\dot{P}_{\mathrm{aux}}^{\mathrm{max}}$	2.25×10^4 Wm^{-3}s^{-1}
$S_{\mathrm{D}}^{\mathrm{max}}$, $S_{\mathrm{T}}^{\mathrm{max}}$	$3 \times S_{\mathrm{D}}^{\mathrm{R}}$, $3 \times S_{\mathrm{T}}^{\mathrm{R}}$
$\dot{S}_{\mathrm{D}}^{\mathrm{max}}$, $\dot{S}_{\mathrm{T}}^{\mathrm{max}}$	3×10^{18}m^{-3}s^{-2}
$I_{\mathrm{coils}}^{\mathrm{max}}$	4 kA

V. Conclusions and Future Work

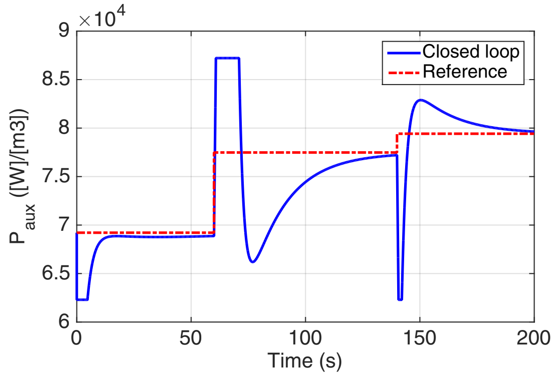
A nonlinear controller that is capable of stabilizing the densities of the different particles and the energy of a burning plasma in a tokamak reactor has been proposed. The control algorithm allows the plasma to be driven between considerably distant working points, rejecting great perturbations



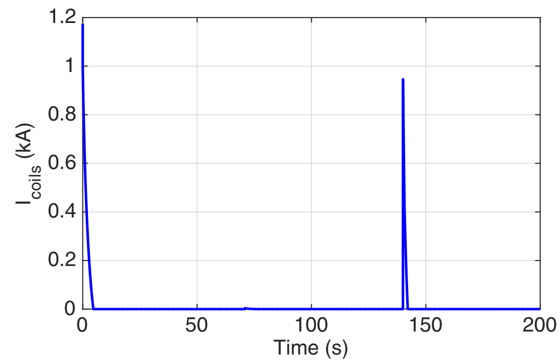
(a) Deuterium injection rate $S_{\mathrm{D}}^{\mathrm{inj}}$



(b) Tritium injection rate $S_{\mathrm{T}}^{\mathrm{inj}}$



(c) Auxiliary power P_{aux}



(d) In-vessel coils current I_{coils}

Fig. 2. Time evolution for (a) deuterium injection rate $S_{\mathrm{D}}^{\mathrm{inj}}$, (b) tritium injection rate $S_{\mathrm{T}}^{\mathrm{inj}}$, (c) auxiliary power P_{aux}, and (d) in-vessel coil current I_{coils}.

in the initial conditions. Such performance is not expected when linear techniques are used.

The inclusion of the in-vessel coils to control the plasma energy improves the effectiveness and widens the applicability of the controller when trying to avoid thermal instabilities. The in-vessel coils are especially effective as an actuator when a short-time release of energy is needed to avoid a thermal excursion. Moreover, the simulation results suggest that the injection of impurities can be avoided with careful control of the in-vessel coils. The drawbacks associated with the use of impurity injection are prevented, allowing for a faster system dynamics.

The model used for τ_E^* has an experimental and physical basis. However, such model needs further development to assure that it correctly represents the actual plasma behavior under different conditions and working points. Deeper research on such model would be a step towards a more efficient and smart burn control. Furthermore, it is necessary to study potential interferences when using the in-vessel coils for other purposes, as plasma vertical position control or suppression of other instabilities, making this actuator unavailable for burn control.

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