Adaptive Nonlinear Burn Control in Tokamak Fusion Reactors

Mark D. Boyer and Eugenio Schuster

Abstract— **There are many challenging control problems critical to the success of burning fusion plasma experiments like ITER. Among them, the most fundamental problem is the control of plasma density and temperature, referred to as the burn condition. While passively stable burn conditions exist, economic and technological constraints may require future commercial fusion reactors to operate at unstable burn conditions. The instability is due to the fact that at low temperatures, the rate of thermonuclear reaction increases as the plasma temperature rises. To stabilize such operating points, it will be essential to have active control of the system. Most existing burn control efforts use control techniques based on linearized models. Such models break down for large perturbations and must be designed around a particular operating point. In this work, we utilize a spatially averaged (zero-dimensional) nonlinear transport model to synthesize a nonlinear feedback controller that can stabilize the burn condition of a fusion reactor. The nonlinear controller guarantees stability of the plasma density and temperature for a much larger range of perturbations than linear designs and is augmented with an adaptive law that guarantees stability despite uncertainty in particle confinement time parameters. A zero-dimensional transport simulation study is presented to show the ability of the controller to bring the system back to the desired equilibrium from a given set of initial perturbations even when there is significant uncertainty in the confinement parameters.**

I. Introduction

Tokamak fusion reactors must be capable of operating for extended periods of time in a burning plasma mode characterized by a high ratio of fusion power to auxiliary power in order to be economically viable as an alternative energy source. For most confinement scalings, passively stable operating points with this characteristic exist, however, they are usually found in a region of high temperature and low density. Due to economic and technological constraints future commercial fusion reactors may be forced to run at low temperature and high density operating points for which the rate of nuclear reaction increases as the plasma temperature rises. Under these conditions, small perturbations in temperature or density will grow without the presence of an active control system. Small increases in temperature lead to thermal excursions in which the system moves to a new, stable equilibrium at a higher temperature, and decreases in temperature can possibly lead to quenching. In either case, disruptive plasma instabilities could be triggered, stopping operation and possibly causing damage to the walls of the confinement vessel.

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The physical and technological feasibility of several methods for controlling the burn condition have been studied over the years. Prior work, including [1], [2], [3], considered auxiliary power, fueling rate, and controlled injection of impurities as actuators to the system. Most existing efforts in the area of burn control make use of just one of theses actuators (single-input control) and linearize the system model to make use of linear control design techniques. In [4], a diagonal multi-input, multi-output linear control scheme was developed for controlling burning plasma kinetics. When tested using nonlinear models, the linear controllers succeed in stabilizing the system against a limited set of perturbations in initial conditions. In our previous work [5], a zero-dimensional nonlinear model involving approximate conservation equations for the energy and the densities of the species was used to synthesize a nonlinear feedback controller for stabilizing the burn condition of a fusion reactor. The controller utilizes all of the previously considered actuators simultaneously, using auxiliary power modulation to prevent quenching, impurity injection to increase radiation losses and stop thermal excursions, and fueling modulation to stabilize the density. Nonlinear burn control using multiple actuators had only been done previously in works using non-model based techniques, like neural networks [6], [7]. The use of nonlinear control techniques removes the operability limits imposed by linearization in other works.

The nonlinear controller designed in our work [5] guarantees a much larger region of attraction than the previous linear controllers. However, the design model assumed an optimal 50:50 mix of deuterium (D) and tritium (T) within the plasma at all times. Because experiments indicate that deuterium and tritium may have different transport properties [8], the D and T systems should be actuated separately to allow for control of the isotopic mix in the core. This scheme, called isotopic fuel tailoring, will be possible on the ITER device, an international collaboration designed to be the first tokamak to explore the burning plasma regime [9], [10]. In [11], we exploited this fueling scheme and used the mix of tritium fuel in the plasma as a method to cool the plasma during thermal excursions. In this way, the use of impurity injection could be avoided. Despite showing good robustness properties, neither of our controllers proposed in [5] or [11] take into account model uncertainties.

The most significant sources of uncertainty in the model are the scaling parameters for the particle confinement times. These parameters are generally not well known and uncertainty can degrade the performance of the nonlinear controller. In this work, we design an adaptive law that estimates the particle confinement times, based on the typical assumption that the particle confinement times scale with the energy confinement time. With the adaptive law in place, stability of the plasma density, tritium ratio, and energy can be guaranteed despite large uncertainties in the particle confinement time scalings.

The paper is organized as follows. In Section II a 0-D ODE model for the dynamics of the densities of alpha-particles, deuterium, tritium, and impurity ions, as well as the energy, is introduced. The desired equilibrium and the control objective is stated in Section III. The design of a nonlinear stabilizing controller and an adaptive law for dealing with particle confinement uncertainty is presented in Section IV. Simulation parameters and results, showing the performance of the controller, are presented in Section V. Finally, conclusions and plans for future work are stated in Section VI.

II. Burning Plasma Model

In this work we use a zero-dimensional model for a burning tokamak plasma which employs approximate energy and particle balance equations. The model is fundamentally the same as that used in [5] and [11]. The deuterium and tritium ion density evolutions are accounted for separately, as the two species are considered to have different transport properties, and a conservation equation for impurity ion density is included. The model is given by the following system of equations

$$\frac{dn_\alpha}{dt} = -\frac{n_\alpha}{\tau_\alpha} + n_D n_T \langle \sigma v \rangle \tag{1}$$

$$\frac{dn_D}{dt} = -\frac{n_D}{\tau_D} - n_D n_T \langle \sigma v \rangle + S_D \tag{2}$$

$$\frac{dn_T}{dt} = -\frac{n_T}{\tau_T} - n_D n_T \langle \sigma v \rangle + S_T \tag{3}$$

$$\frac{dE}{dt} = -\frac{E}{\tau_E} + Q_\alpha n_D n_T \langle \sigma v \rangle - P_{rad} + P_{aux} \tag{4}$$

$$\frac{dn_I}{dt} = -\frac{n_I}{\tau_I} + S_I \tag{5}$$

where n_α, n_D, n_T, and n_I are the alpha-particle, deuterium, tritium, and impurity densities, respectively, and E is the energy. Parameters τ_α, τ_D, τ_T, τ_E, τ_I are the (state-dependent) alpha-particle, deuterium, tritium, energy, and impurity confinement times, respectively. The control inputs are the deuterium, tritium, and impurity injection rates, given by S_D, S_T, and S_I, as well as the auxiliary heating, P_{aux}. This approximate model implies that the alpha-particles slow down instantaneously and deposit their energy ($Q_\alpha = 3.52$ MeV) in the flux surface in which they are born, which is a reasonable approximation for reactor-size tokamaks.

The DT reactivity $\langle \sigma v \rangle$ is a highly nonlinear, positive, and bounded function of the plasma temperature T and is calculated by

$$\langle \sigma v \rangle = exp\left(\frac{a_1}{T^r} + a_2 + a_3 T + a_4 T^2 + a_5 T^3 + a_6 T^4\right) \tag{6}$$

where the parameters a_i and r are taken from [12].

In this work, the radiation loss P_{rad} is approximated as

$$P_{rad} = P_{brem} = A_b Z_{eff} n_e^2 \sqrt{T} \tag{7}$$

where $A_b = 5.5 \times 10^{-37}$ $\mathrm{Wm^3}/\sqrt{\mathrm{keV}}$ is the bremsstrahlung radiation coefficient, Z_{eff} is the effective atomic number, and n_e is the electron density. No explicit evolution equation is provided for the electron density as it can be obtained from the neutrality condition $n_e = n_D + n_T + 2n_\alpha + Z_I n_I$. The effective atomic number, plasma density, and temperature are given by

$$Z_{eff} = \sum_i \frac{n_i Z_i^2}{n_e} = \frac{n_D + n_T + 4n_\alpha + Z_I^2 n_I}{n_e} \tag{8}$$

$$n = n_\alpha + n_D + n_T + n_I + n_e \tag{9}$$

$$= 2n_D + 2n_T + 3n_\alpha + (Z_I + 1)n_I \tag{10}$$

$$T = \frac{2}{3}\frac{E}{n} \tag{11}$$

where Z_i is the atomic number of the different ions. The energy confinement time scaling used in this work is ITER90H-P [13] because it allows for performance comparison with previous work, however, this choice only affects the simulation study. The controller design is independent of the scaling used. The scaling used for simulations is

$$\tau_E = f_\tau 0.082 I^{1.02} R^{1.6} B^{0.15} A_i^{0.5} \kappa_\chi^{-0.19} P^{-0.47} = f_\tau k A_i^{0.5} P^{-0.47} \tag{12}$$

The scale factor f_τ depends on the confinement mode and is determined by comparing the net plasma heating power P to the L-H transition power (low confinement (L-mode) to high confinement (H-mode) transition power). For the simulations studied, the system remains in H-mode, for which we use $f_\tau = 0.85$. If a transition to L-mode were to occur, f_τ would be reduced to reflect the transition. Parameters I, R, B, κ_χ, are assumed to be kept constant by a magnetic control algorithm, such that they can be collapsed into a single constant, k. The isotopic number A_i is given by the expression $A_i = 3\gamma + 2(1-\gamma) = \gamma + 2$ with γ being the tritium fraction. This is the fraction of hydrogenic ions in the plasma that are tritium ions and can be expressed as $\gamma = n_T/(n_D + n_T)$. The net plasma heating power P is defined as $P = P_{fusion} - P_{rad} + P_{aux}$. The fusion power is given by

$$P_{fusion} = Q_\alpha n_D n_T \langle \sigma v \rangle = Q_\alpha \gamma (1-\gamma) n_H^2 \langle \sigma v \rangle \tag{13}$$

where $n_H = n_D + n_T$ is the total hydrogenic density. We note here that the relationship between fusion power and tritium fraction γ is parabolic with a maximum fusion power at 0.5, making it important to regulate the tritium fraction about this point for reactor efficiency.

The confinement times for the different species are scaled with the energy confinement time τ_E as

$$\tau_\alpha = k_\alpha \tau_E,\ \tau_D = k_D \tau_E,\ \tau_T = k_T \tau_E\ \tau_I = k_I \tau_E \tag{14}$$

In Section IV, the parameters k_α, k_D, k_T, and k_I are estimated online using an adaptive law. The reactor parameters used in this simulation are given in Table I.

TABLE I
REACTOR PARAMETERS

Symbol	Description	Value
I	Plasma current	22.0 MA
R	Major radius	6.0 m
a	Minor radius	2.15 m
B	Magnetic field	4.85 T
κ_χ	Elongation at the x-point	2.2
β_{max}	Beta limit	2.5I/aB=5.3%
V	Plasma volume	1100 m^3

III. CONTROL OBJECTIVE AND APPROACH

The equilibria of the dynamic equations (1) through (5) give the possible steady-state operating points of the reactor. The equilibrium values of the energy $\bar{E}$, the density variables $\bar{n}_\alpha$, $\bar{n}_D$, $\bar{n}_T$, the fueling source terms $\bar{S}_D$, $\bar{S}_T$, and the auxiliary heating $\bar{P}_{aux}$, are determined by solving the nonlinear algebraic equations obtained by setting the left side of Eqs. (1) through (5) to zero when three of the plasma parameters, such as temperature T, β[1] and γ, for example, are chosen arbitrarily, i.e., we solve

$$0 = -\frac{\bar{n}_\alpha}{\bar{\tau}_\alpha} + \bar{n}_D\bar{n}_T\langle\bar{\sigma v}\rangle \quad (15)$$

$$0 = -\frac{\bar{n}_D}{\bar{\tau}_D} - \bar{n}_D\bar{n}_T\langle\bar{\sigma v}\rangle + \bar{S}_D \quad (16)$$

$$0 = -\frac{\bar{n}_T}{\bar{\tau}_T} - \bar{n}_D\bar{n}_T\langle\bar{\sigma v}\rangle + \bar{S}_T \quad (17)$$

$$0 = -\frac{\bar{E}}{\bar{\tau}_E} + Q_\alpha\bar{n}_D\bar{n}_T\langle\bar{\sigma v}\rangle - \bar{P}_{rad} + \bar{P}_{aux} \quad (18)$$

$$0 = \bar{n}_I \quad (19)$$

Ideally, the equilibrium sources $\bar{P}_{aux}$, $\bar{S}_D$, and $\bar{S}_T$ would force the plasma to the desired equilibrium ($\bar{T}$, $\bar{\beta}$, $\bar{\gamma}$), however, certain equilibria are unstable and require active control to be maintained. Additionally, because the particle confinement parameters k_α, k_D, k_T, and k_I are uncertain, the calculated equilibrium sources may not achieve the desired conditions, even for stable equilibria. The objective, therefore, is to design a controller that stabilizes $\bar{T}$, $\bar{\beta}$, and $\bar{\gamma}$ despite uncertainty in the particle confinement parameters. It is assumed that all of the states are available from either measurement or estimation. P_{aux} is used to stabilize the energy of the system during negative perturbations, however, since P_{aux} cannot be reduced below zero, large positive perturbations in temperature require the use of another actuator. In this work, controlled injection of impurities is used to increase radiation losses and stabilize such excursions. The remaining actuators, S_D and S_T, are used to stabilize the plasma density and tritium ratio. An adaptive law is used to overcome uncertainty in the particle confinement parameters.

By defining the deviations from the desired equilibrium values as $\tilde{n}_\alpha = n_\alpha - \bar{n}_\alpha$, $\tilde{n}_D = n_D - \bar{n}_D$, $\tilde{n}_T = n_T - \bar{n}_T$, $\tilde{n}_I = n_I$, and $\tilde{E} = E - \bar{E}$, the dynamic equations for the deviations can be written as

$$\frac{d\tilde{n}_\alpha}{dt} = -\frac{\tilde{n}_\alpha}{\tau_\alpha} - \frac{\bar{n}_\alpha}{\tau_\alpha} + S_\alpha \quad (20)$$

$$\frac{d\tilde{n}_D}{dt} = -\frac{\tilde{n}_D}{\tau_D} - \frac{\bar{n}_D}{\tau_D} - S_\alpha + S_D \quad (21)$$

$$\frac{d\tilde{n}_T}{dt} = -\frac{\tilde{n}_T}{\tau_T} - \frac{\bar{n}_T}{\tau_T} - S_\alpha + S_T \quad (22)$$

$$\frac{d\tilde{E}}{dt} = -\frac{\tilde{E}}{\tau_E} - \frac{\bar{E}}{\tau_E} + Q_\alpha S_\alpha - P_{rad} + P_{aux} \quad (23)$$

$$\frac{d\tilde{n}_I}{dt} = -\frac{\tilde{n}_I}{\tau_I} + S_I \quad (24)$$

where the nonlinear α generation term has been written as

$$S_\alpha(E, n_D, n_T, n_\alpha) = n_D n_T\langle\sigma v\rangle = \gamma(1-\gamma)n_H^2\langle\sigma v\rangle \quad (25)$$

to simplify the presentation. Recall from equations (6) and (11) that $\langle\sigma v\rangle$ is a function of E, n_D, n_T, n_α and n_I.

IV. ADAPTIVE NONLINEAR CONTROLLER DESIGN

The design begins by looking for a control which stabilizes $\tilde{E}$. By satisfying the condition

$$Q_\alpha S_\alpha - P_{rad} + P_{aux} = \frac{\bar{E}}{\tau_E} \quad (26)$$

equation (23) is reduced to

$$\frac{d\tilde{E}}{dt} = -\frac{\tilde{E}}{\tau_E} \quad (27)$$

and the $\tilde{E}$ subsystem is then exponentially stable since $\tau_E > 0$. Condition (26) is met by modulating the impurity density n_I and the auxiliary heating P_{aux}. Modulation of the impurity density is done through altering the impurity injection rate S_I. We calculate the auxiliary power needed to satisfy condition (26) as

$$P_{aux} = \frac{\bar{E}}{\tau_E} - Q_\alpha\gamma^*(1-\gamma^*)n_H^2\langle\sigma v\rangle + P_{rad} \quad (28)$$

If this value is negative, we set $P_{aux} = 0$. For such cases, condition (26) cannot be met by modulation of auxiliary heating alone, and impurity injection is needed. The desired impurity density n_I^* is calculated as the value satisfying

$$A_b Z_{eff} n_e^2\sqrt{T} = -\frac{\bar{E}}{\tau_E} + Q_\alpha S_\alpha \quad (29)$$

where we note that Z_{eff}, n_e, and T are functions of n_I.

Based on the control objectives and the previous steps, we now have desired reference values for the energy, $\bar{E}$, the tritium fraction, $\bar{\gamma}$, the plasma density, $\bar{n}$, and the impurity density n_I^*, along with a controller request for the auxiliary heating P_{aux}. We now seek a stabilizing control law using the fueling terms S_D, S_T, and S_I. We begin by defining

$$\hat{n}_I = n_I - n_I^* \quad (30)$$

$$f(\hat{n}_I, \tilde{E}, \tilde{n}_\alpha, \tilde{n}_D, \tilde{n}_T) = -\frac{\bar{E}}{\tau_E} + Q_\alpha S_\alpha - P_{rad} + P_{aux} \quad (31)$$

$$f(n_I^*, \tilde{E}, \tilde{n}_\alpha, \tilde{n}_D, \tilde{n}_T) = 0 \quad (32)$$

[1]The plasma β is the ratio of plasma pressure to magnetic pressure $\beta = \frac{knT}{(B^2/2\mu_0)}$ where B is the magnetic field strength, μ_0 is the permeability of free space, and k is the Boltzmann constant.

where the last relation is a consequence of our choice of P_{aux} and n_I^* in (28) and (29). We can then write $f = \hat{n}_I \phi$, where ϕ is a continuous function. This allows us to rewrite (23) as

$$\frac{d\tilde{E}}{dt} = -\frac{\tilde{E}}{\tau_E} + \hat{n}_I \phi \tag{33}$$

Along with the nonlinear controller, we seek an adaptive scheme for estimating the particle confinement scaling constants. To facilitate the design process, we define a matrix of the nominal parameters

$$\theta^* = \begin{bmatrix} 1/k_\alpha & 1/k_D & 1/k_T & 1/k_I \end{bmatrix}^T \tag{34}$$

We define our estimate of the confinement parameters as $\hat{\theta}$ and the error of the estimate as $\tilde{\theta} = \hat{\theta} - \theta^*$. Next, we use the elements of θ^* to write the dynamic equations for the to-be-controlled variables $\hat{n}_I$, $\tilde{\gamma}$, and $\tilde{n}$. We can write the dynamics of $\hat{n}_I$ by noting that

$$\dot{n}_I = \dot{\hat{n}}_I + \dot{n}_I^* = -\theta_4^* \frac{n_I}{\tau_E} + S_I \tag{35}$$

$$\dot{\hat{n}}_I = -\theta_4^* \frac{n_I}{\tau_E} + S_I - \dot{n}_I^* \tag{36}$$

Recalling the definition of the tritium ratio, $\gamma = n_T/n_H$, we can write the equation governing its dynamics as

$$\dot{\gamma} = \dot{\tilde{\gamma}} = \frac{\dot{n}_T n_H - n_T \dot{n}_H}{n_H^2} = \frac{\dot{n}_T}{n_H} - \gamma \frac{\dot{n}_H}{n_H} \tag{37}$$

We recall (21) and (22) to write

$$\dot{n}_T = \dot{\tilde{n}}_T = -\theta_3^* \frac{n_T}{\tau_E} - S_\alpha + S_T \tag{38}$$

$$\dot{n}_H = \dot{\tilde{n}}_H = \dot{\tilde{n}}_T + \dot{\tilde{n}}_D = -\theta_3^* \frac{n_T}{\tau_E} - \theta_2^* \frac{n_D}{\tau_E} - 2S_\alpha + S_D + S_T \tag{39}$$

$$\begin{aligned} \dot{\tilde{\gamma}} = &\frac{1}{n_H} \Bigg[-\theta_3^* \frac{n_T}{\tau_T} - S_\alpha + S_T \\ &- \gamma \left(-\theta_3^* \frac{n_T}{\tau_E} - \theta_2^* \frac{n_D}{\tau_E} - 2S_\alpha + S_D + S_T \right) \Bigg] \end{aligned} \tag{40}$$

In addition, we recall (20) and (24) to write

$$\dot{n} = \dot{\tilde{n}} = 3\dot{\tilde{n}}_a + 2\dot{\tilde{n}}_T + 2\dot{\tilde{n}}_D + (Z_I+1)\dot{\tilde{n}}_I \tag{41}$$

$$\begin{aligned} = &-3\theta_1^* \frac{n_a}{\tau_E} - 2\theta_3^* \frac{n_T}{\tau_E} - 2\theta_2^* \frac{n_D}{\tau_E} - (Z_I+1)\theta_4^* \frac{n_I}{\tau_E} \\ &- S_\alpha + 2S_D + 2S_T + (Z_I+1)S_I \end{aligned} \tag{42}$$

We take as a Lyapunov function

$$V = \frac{k_1^2 \tilde{E}^2 + k_2^2 \tilde{\gamma}^2 + \hat{n}_I^2 + \tilde{n}^2}{2} + \tilde{\theta}^T \frac{\Gamma^{-1}}{2} \tilde{\theta} \tag{43}$$

where $k_1 = 10^{15}$, $k_2 = 10^{20}$ (recall that $\tilde{E} = O(10^5)$, $\tilde{\gamma} = O(10^{-1})$, and $\tilde{n} = O(10^{20})$) and Γ is a positive definite matrix. The derivative can be written as

$$\dot{V} = k_1^2 \tilde{E}\dot{\tilde{E}} + k_2^2 \tilde{\gamma}\dot{\tilde{\gamma}} + \tilde{n}\dot{\tilde{n}} + \hat{n}_I \dot{\hat{n}}_I + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \tag{44}$$

$$\begin{aligned} = &-\frac{k_1^2 \tilde{E}^2}{\tau_E} + \frac{k_2^2 \tilde{\gamma}}{n_H} \Bigg[-\theta_3^* \frac{n_T}{\tau_E} - S_\alpha + S_T \\ &- \gamma \left(-\theta_3^* \frac{n_T}{\tau_E} - \theta_2^* \frac{n_D}{\tau_E} - 2S_\alpha + S_D + S_T \right) \Bigg] \\ &+ \tilde{n} \Big[-3\theta_1^* \frac{n_a}{\tau_E} - 2\theta_3^* \frac{n_T}{\tau_E} - 2\theta_3^* \frac{n_D}{\tau_E} - (Z_I+1)\theta_4^* \frac{n_I}{\tau_E} \\ &- S_\alpha + 2S_D + 2S_T + (Z_I+1)S_I \Big] \\ &+ \hat{n}_I \left[k_1^2 \tilde{E}\phi - \theta_4^* \frac{n_I}{\tau_E} + S_I - \dot{n}_I^* \right] + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \end{aligned} \tag{45}$$

Since the the nominal parameter θ^* is uncertain, we use the certainty equivalence approach and take as control laws

$$S_I = \hat{\theta}_4 \frac{n_I}{\tau_E} + \dot{n}_I^* - k_1^2 \tilde{E}\phi - K_I \hat{n}_I \tag{46}$$

$$\begin{aligned} S_D = &\frac{1}{2} \Bigg[3\hat{\theta}_1 \frac{n_a}{\tau_E} + 2\hat{\theta}_3 \frac{n_T}{\tau_E} + 2\hat{\theta}_2 \frac{n_D}{\tau_E} + S_\alpha - 2S_T \\ &- (Z_I+1) \left(S_I - \hat{\theta}_4 \frac{n_I}{\tau_E} \right) - K_n \tilde{n} \Bigg] \end{aligned} \tag{47}$$

$$\begin{aligned} S_T = &-K_\gamma \tilde{\gamma} + \theta_3^* \frac{n_T}{\tau_E} + S_\alpha + \gamma \Bigg(\theta_1^* \frac{3n_a}{2\tau_E} - \frac{3}{2} S_\alpha \\ &- \frac{(Z_I+1)}{2} \left(S_I - \theta_4^* \frac{n_I}{\tau_E} \right) - \frac{K_n \tilde{n}}{2} \Bigg) \end{aligned} \tag{48}$$

where $\hat{\theta}$ is the current estimate of the confinement parameters and the constants K_I, K_n, and K_γ are positive. Given exact knowledge of the nominal parameters, i.e., $\hat{\theta} = \theta^*$, this choice would result in

$$\dot{V} = -\frac{k_1^2 \tilde{E}^2}{\tau_E} - K_\gamma \frac{k_2^2 \tilde{\gamma}^2}{n_H} - K_n \tilde{n}^2 - K_I \hat{n}_I^2 \leq 0 \tag{49}$$

which would guarantee that the energy, plasma density, and tritium fraction subsystems were stabilized, and that the impurity density could converge to n_I^*. In general, there will be some error in the estimate of the confinement parameters, such that after substituting (46), (47), and (48), and gathering terms, the expression (45) becomes

$$\begin{aligned} \dot{V} = &-\frac{k_1^2 \tilde{E}^2}{\tau_E} - K_\gamma \frac{k_2^2 \tilde{\gamma}^2}{n_H} - K_n \tilde{n}^2 - K_I \hat{n}_I^2 + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \\ &3\tilde{n}\tilde{\theta}_1 \frac{n_a}{\tau_E} + \left[2\tilde{n} - (\gamma - 1) \frac{k_2^2 \tilde{\gamma}}{n_H} \right] \tilde{\theta}_3 \frac{n_T}{\tau_E} \\ &+ \left[2\tilde{n} - \frac{k_2^2 \tilde{\gamma}}{n_H} \gamma \right] \tilde{\theta}_2 \frac{n_D}{\tau_E} + [\tilde{n}(Z_I+1) + \hat{n}_I] \tilde{\theta}_4 \frac{n_I}{\tau_E} \end{aligned} \tag{50}$$

We take as an adaptive law:

$$\dot{\tilde{\theta}} = \frac{1}{\tau_E} \Gamma \begin{bmatrix} -3\tilde{n} n_\alpha \\ -\left[2\tilde{n} - \frac{k_2^2 \tilde{\gamma}}{n_H} \gamma \right] n_D \\ -\left[2\tilde{n} - (\gamma - 1) \frac{k_2^2 \tilde{\gamma}}{n_H} \right] n_T \\ -[\tilde{n}(Z_I+1) + \hat{n}_I] n_I \end{bmatrix} \tag{51}$$

which reduces (50) to

$$\dot{V} = -\frac{k_1^2 \tilde{E}^2}{\tau_E} - K_\gamma \frac{k_2^2 \tilde{\gamma}^2}{n_H} - K_n \tilde{n}^2 - K_I \hat{n}_I^2 \leq 0 \tag{52}$$

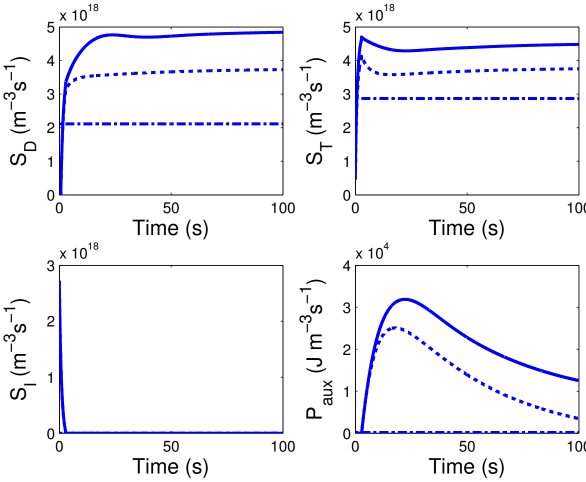


Fig. 1. Actuators during open loop (dash-dot), closed loop (dot), and adaptive (solid) cases.

and guarantees that the n, E, n_I, and γ are all driven to the desired values. Note that this scheme does not guarantee that the parameter estimates converge to the actual values, only that the states are stabilized.

V. Simulation Results

In this section we show, through a simulation study, the ability of the controller to stabilize an equilibrium characterized by $\bar{T} = 8.2$ keV, $\bar{\beta} = 3\%$, and $\bar{\gamma} = 0.5$. We consider the nominal confinement parameters and initial estimates given in Table II. We utilize the initial estimates of the confinement parameters to synthesize the closed loop controller as well as to calculate the open loop inputs ($\bar{S}_D$, $\bar{S}_T$, $\bar{S}_I$, $\bar{P}_{aux}$). We perform one open-loop simulation and two closed-loop simulations: one with $\Gamma = 0$, i.e., without adaptation, and one with $\Gamma = .5\mathbf{I}$, i.e., with adaptation. The simulations all use the initial perturbations $T(0) = 1.43\bar{T}$, $\beta(0) = 1.20\bar{\beta}$, and $\gamma(0) = 0.88\bar{\gamma}$.

Figure 1 compares the inputs during the simulations. Note how the impurity injection and auxiliary power work in tandem to control the system in closed loop. Initially, the controller determines that an increase in radiation losses is necessary and injects impurities. After the excursion, the impurity density within the plasma slowly decays and auxiliary heating is used to overcome the excess radiation losses and fuel dilution caused by the impurities. Figure 2 shows how the estimates of the elements of θ evolve over time compared with the actual values θ^*. The adaptive controller does not guarantee that the estimates converge to the actual values, which is reflected in the results. We note, however, that the control objective is to reach the desired equilibrium, not to identify model parameters. Figure 3 shows the response of the variables of interest (T, n, γ, β), during the three simulation cases. It can be seen that in open loop, the system moves far from the desired equilibrium. This is mainly because the desired equilibrium is unstable, but also a result of the error in the estimated particle confinement parameters used to calculate the open loop inputs. This illustrates the importance of implementing feedback control of

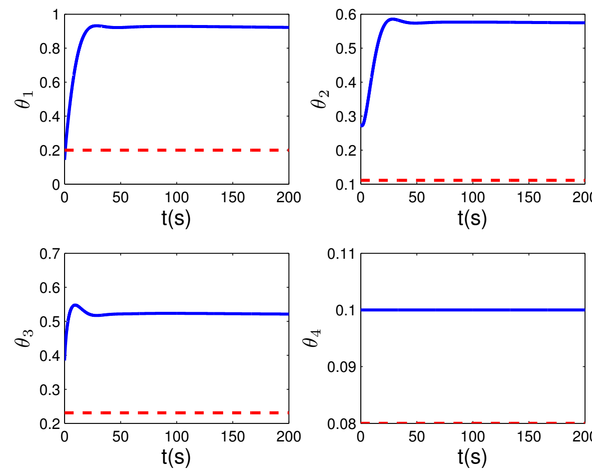


Fig. 2. Adaptive estimation of θ (blue-solid) and θ^* (red-dashed).

TABLE II

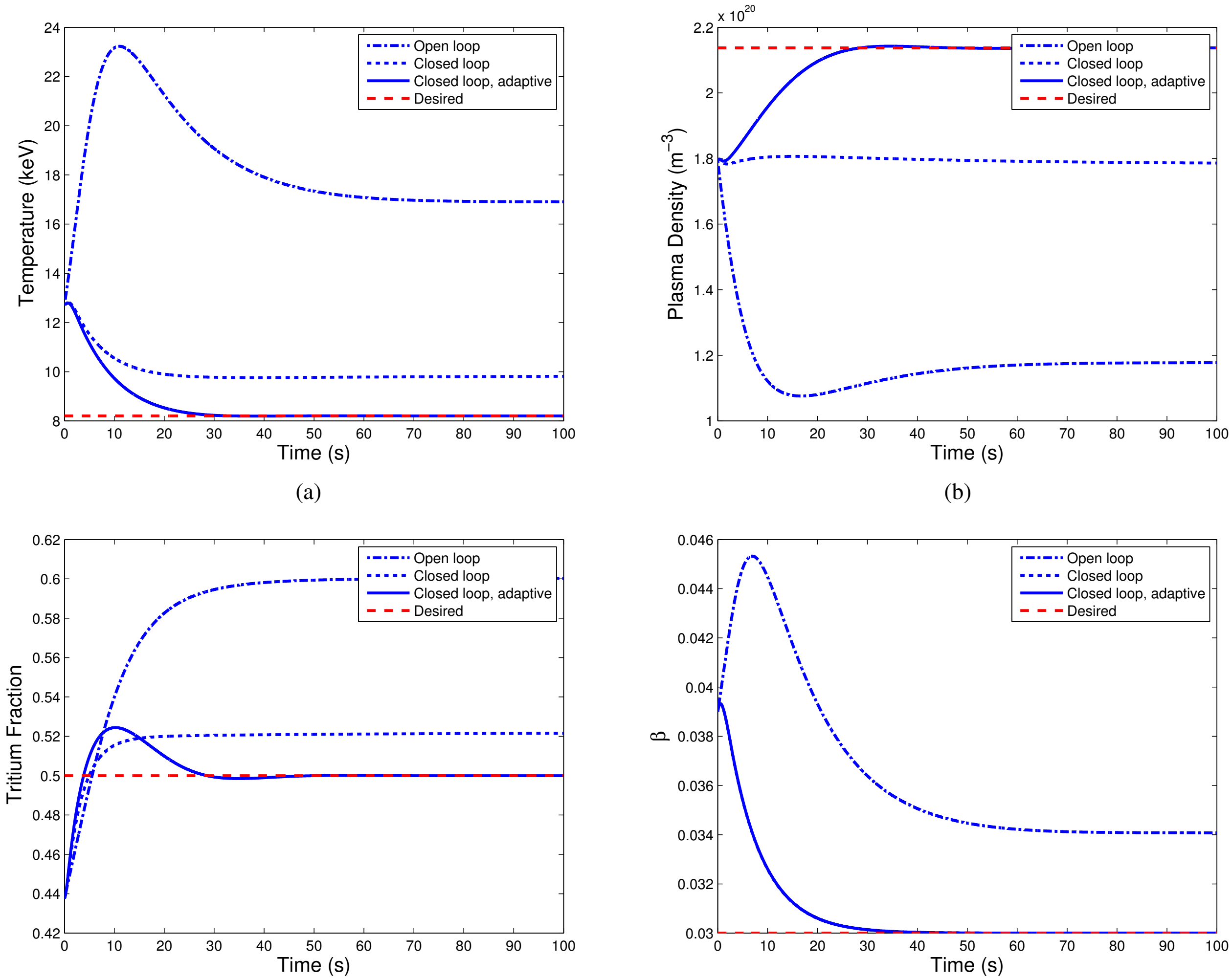
Particle Confinement Parameters

Parameter	Nominal	Initial Estimate
k_α	9.8	7
k_D	1.44	3.6
k_T	1.56	2.6
k_I	8	10

the burn condition, even when operating at stable equilibria. The figures also show that in the closed loop simulation without the adaptive law activated, the controller improves upon the plasma performance, but still does not reach the desired equilibrium in steady state. This is due to the particle confinement estimation error. Finally, when the adaptive law is active, it can be seen that the desired equilibrium is stabilized despite the instability of the equilibrium, the initial perturbations, and the uncertainty in confinement parameters.

VI. Conclusions and Future Work

We have presented a nonlinear controller that is capable of stabilizing the temperature, density, and tritium ratio of a burning tokamak plasma despite uncertainty in the particle confinement parameters. By using the full nonlinear model, the controller can deal with a larger set of perturbations in initial conditions than previous linear controllers and the multi-input scheme allows it to reject initial conditions leading to both thermal excursion and quenching. The adaptive law guarantees stability for arbitrarily large uncertainty in confinement parameters. In addition, the effectiveness of the controller does not depend on whether the operating point is an ignition or subignition point. Since the nonlinear controller depends parametrically on the equilibrium point, it can be used to drive the system from one working point to another, allowing in this way for changes in power and other plasma parameters without the need for scheduled controllers. The system was simulated in open loop, closed loop without adaptation, and closed loop with adaptation to show the need for and the effectiveness of the proposed scheme. The simulations show that the adaptive controller can effectively force the system back to the desired states despite uncertain particle confinement times.



(a) (b) (c) (d)

Fig. 3. Comparison of (a) temperature, (b) plasma density, (c) tritium fraction, and (d) β for open loop, closed loop, and adaptive cases.

This approach could be used for any other energy confinement time scalings (12) and is not restricted to the ITER scaling used here. Note that the expressions for confinement time used here were derived experimentally under time-stationary states. Thus, it is not clear whether the empirical scaling can be safely applied for large excursions away from equilibrium (i.e., when $(1/\tilde{E})(d\tilde{E}/dt > 1/\tau_E)$. In this case, an adaptive scheme for estimating τ_E could be developed or perhaps a one-dimensional model could be used to incorporate plasma profile and transport information during transients. Use of such a model will be a step towards kinetic profile control in burning plasmas, an issue with implications for other fusion control problems, like transport, improvement of energy confinement, and MHD stability.

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