Nonlinear Burn Control in Fusion Reactors

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Abstract**-- Control of plasma density and temperature magnitudes, as well as their profiles, are among the most fundamental problems in fusion reactors. Unfortunately, the economy of fusion reactors often requires the reactor to operate under conditions in which the rate of thermonuclear reaction increases as the plasma temperature rises. In this thermally unstable zone, an active control system is necessary to stabilize the thermonuclear reaction. Existing efforts use control techniques for linear models. In this work, a zero-dimensional nonlinear model involving approximate conservation equations for the energy and the densities of the species was used to synthesize a nonlinear feedback controller for stabilizing the burn condition of a fusion reactor. The controller makes use simultaneously of the modulation of auxiliary power, the modulation of fueling rate and the controlled injection of impurities as actuators. A computer simulation study was performed to show the capability of the controller and compare it with previous linear controllers.**

I. Introduction

In order to be commercially competitive, a fusion reactor needs to run long periods of time in a stable burning plasma mode at working points which are characterized by a high Q, where Q is the ratio of fusion power to auxiliary power. Active burn control is often required to maintain these near-ignited or ignited conditions. Although operating points with these characteristics that are inherently stable exist for most confinement scalings, they are found in a region of high temperature and low density. Unfortunately, economical and technological constraints make these operating points unattractive and require the fusion reactor to operate in a zone of low temperature and high density where the thermonuclear reaction is inherently thermally unstable. In this thermally unstable zone, a small increase of temperature leads to an increase of power that results *in thermal excursion*. Although the excursion reaches a stable uneconomical working point at a higher temperature, the plasma can be led to beta or density limit disruptions before reaching this point. On the other hand, a small decrease of temperature leads to a decrease of power and *quenching*. Even during a quenching, a disruptive instability can be reached, causing wall damage.

The prior efforts on active burn control have led to a consensus in the community that the control system should take into account the nonlinear nature of the dynamic model, be robust against uncertainties of some parameters of the dynamic model and operate under different operating conditions allowing change in fusion power.

Over the years, the physical and technological feasibility of different methods for controlling the burn condition have been studied [1], [2], [3]. In these studies, mainly three different types of actuation have been considered: modulation of auxiliary power, modulation of fueling rate and controlled injection of impurities.

The common denominator of existing works is the approximation of the nonlinear model of the fusion reactor by a linearized one and in most of the cases the utilization of only one among the actuation concepts (single-input control). To expand operability, we are seeking a systematic procedure for synthesis of burn controllers that are able to stabilize the system against large initial conditions, can work as well for suppressing thermal excursions as for preventing quenches, can operate at subignition or ignition points indistinctly, show robustness against uncertainties in parameters of the model such as the confinement times of the species, can drive the system from an operating point to another and can change the fusion power during the reactor operation. Such controllers should be based on a full nonlinear model and should make use simultaneously of all the potential actuators: auxiliary power, refueling rate and impurities injection.

The paper is organized as follows. In Section 2 a zero-dimensional model for the fusion reactor is described. The control objectives are stated in Section 3. A nonlinear feedback control law that achieves stabilization of the deviation state variables is presented in Section 4. In Section 5, a detailed simulation study is provided. Finally, the conclusions and some suggestions about future work are presented in Section 6.

II. Model

In this work we use a zero-dimensional model for a fusion reactor that employs approximate particle and energy balance equations. This is fundamentally the same model used by Bamieh, Hui and Miley and [5] but we introduce a new equation which allows the presence of impurities in the reactor. The alpha-particle balance is given by

$$\frac{dn_\alpha}{dt} = -\frac{n_\alpha}{\tau_\alpha} + \left(\frac{n_{DT}}{2}\right)^2 \langle \sigma v \rangle \qquad (1)$$

where n_α and n_{DT} are the alpha and deuterium-tritium *(DT)* densities respectively, and τ_α is the confinement time for the alpha particles. The deuterium-tritium *(DT)* fuel particle balance is given by

$$\frac{dn_{DT}}{dt} = -\frac{n_{DT}}{\tau_{DT}} - 2\left(\frac{n_{DT}}{2}\right)^2 \langle \sigma v \rangle + S \qquad (2)$$

where S is the refueling rate (input) and τ_{DT} is the confinement time for the fuel particles. The impurity presence is determined by the balance equation

$$\frac{dn_I}{dt} = -\frac{n_I}{\tau_I} + S_I \tag{3}$$

where n_I is the impurity density, τ_I is the confinement time for the impurity particles and S_I is the impurity injection rate (input). The energy balance is given by

$$\frac{dE}{dt} = -\frac{E}{\tau_E} + \left(\frac{n_{DT}}{2}\right)^2 \langle\sigma v\rangle Q_\alpha - P_{rad} + P_{aux} \tag{4}$$

where E is the plasma energy, P_{aux} is the auxiliary power (input), Q_α=3.52 MeV is the energy of the alpha particles and the radiation loss P_{rad} is given by $P_{rad} = A_b Z_{eff} n_e^2 \sqrt{T}$ where $A_b = 5.5\ 10^{-37} Wm^3/\sqrt{KeV}$ is the Bremsstrahlung radiation coefficient. The DT reactivity $\langle\sigma v\rangle$ is a highly nonlinear, positive and bounded function of the plasma temperature T given by

$$\langle\sigma v\rangle = \exp\left\{\frac{a_1}{T^r} + a_2 + a_3 T + a_4 T^2 + a_5 T^3 + a_6 T^4\right\} \tag{5}$$

and its constant parameters a_i and r are taken from [6]. No explicit evolution equation is provided for the electron density n_e since we can obtain it from the neutrality condition $n_e = n_{DT} + 2n_\alpha + Z_I n_I$, whereas the effective atomic number Z_{eff}, the total density and the energy are written as

$$Z_{eff} = \frac{\sum_i n_i Z_i^2}{n_e} = \frac{n_{DT} + 4n_\alpha + z_i^2 n_I}{n_e} \tag{7}$$

$$n = n_\alpha + n_{DT} + n_I + n_e = 2n_{DT} + 3n_\alpha + (z_I + 1)n_I \tag{8}$$

$$E = \frac{3}{2} nT \Rightarrow T = \frac{2}{3}\frac{E}{2n_{DT} + 3n_\alpha + (z_I + 1)n_I} \tag{9}$$

The energy confinement scaling used in this work is ITER90H-P [7] because it allows the comparison with previous linear controllers based on this scaling. However, it will be clear from the synthesis procedure that the results can be extended to newer scalings.

$$\tau_E = f\,0.082\, I^{1.02} R^{1.6} B^{0.15} A_i^{0.5} \kappa_\chi^{-0.19} P^{-0.47} = kP^{-0.47} \tag{10}$$

where the isotopic number A_i is 2.5 for the 50:50 DT mixture, k is a constant that depends on the ITER machine parameters which are defined in table I and the factor scale f which in turn depends on the confinement mode. The net plasma heating power P is defined as

$$P = \left(\frac{n_{DT}}{2}\right)^2 \langle\sigma v\rangle Q_\alpha - P_{rad} + P_{aux} \tag{11}$$

The confinement times for the different species are scaled with the energy confinement time τ_E as

$$\tau_\alpha = k_\alpha \tau_E, \quad \tau_\alpha = k_\alpha \tau_E, \quad \tau_I = k_I \tau_E. \tag{12}$$

III. Control Objective

The possible operating points of the reactor are given by the equilibria of the dynamic equations. The density state variables $\overline{n}_\alpha$, $\overline{n}_{DT}$, $\overline{n}_I = 0$, energy state variable $\overline{E}$ and inputs $\overline{P}_{aux}$, $\overline{S}$, $\overline{S}_I = 0$ at the equilibrium, are calculated as solutions of the nonlinear algebraic equations obtained by setting the left hand sides in (1)-(4) to zero when two of the plasma parameters such as T and β, for example, are chosen arbitrarily.

Defining the deviations from the desired equilibrium values as $\widetilde{n}_\alpha = n_\alpha - \overline{n}_\alpha$, $\widetilde{n}_{DT} = n_{DT} - \overline{n}_{DT}$, $\widetilde{n}_I = n_I - \overline{n}_I = n_I$, $\widetilde{E} = E - \overline{E}$, $\widetilde{P}_{aux} = P_{aux} - \overline{P}_{aux}$, $\widetilde{S} = S - \overline{S}$, $\widetilde{S}_I = S_I - \overline{S}_I = S_I$, we write the dynamic equations for the deviations as

$$\frac{d\widetilde{n}_\alpha}{dt} = -\frac{\widetilde{n}_\alpha}{\tau_\alpha} + \left(\frac{\widetilde{n}_{DT}}{2}\right)^2 \langle\sigma v\rangle + \frac{1}{2}\widetilde{n}_{DT}\overline{n}_{DT}\langle\sigma v\rangle + u_\alpha \tag{13}$$

$$\frac{d\widetilde{n}_{DT}}{dt} = -\frac{\widetilde{n}_{DT}}{\tau_{DT}} - \left[2\left(\frac{\widetilde{n}_{DT}}{2}\right)^2 + \widetilde{n}_{DT}\overline{n}_{DT}\right]\langle\sigma v\rangle + \widetilde{S} + u_{DT} \tag{14}$$

$$\frac{d\widetilde{n}_I}{dt} = -\frac{\widetilde{n}_I}{\tau_I} + S_I \tag{15}$$

$$\frac{d\widetilde{E}}{dt} = -\frac{\widetilde{E}}{\tau_E} - \left[\frac{\overline{E}}{\tau_E} - \left[\left(\frac{n_{DT}}{2}\right)^2 \langle\sigma v\rangle Q_\alpha + u\right]\right] \tag{16}$$

where

$$u_\alpha = -\frac{\overline{n}_\alpha}{\tau_\alpha} + \left(\frac{\overline{n}_{DT}}{2}\right)^2 \langle\sigma v\rangle$$

$$u_{DT} = -\frac{\overline{n}_{DT}}{\tau_{DT}} - 2\left(\frac{\overline{n}_{DT}}{2}\right)^2 \langle\sigma v\rangle + \overline{S}$$

$$u = P_{aux} - P_{rad}$$

The control objective is to drive the initial perturbations in $\widetilde{n}_\alpha$, $\widetilde{n}_{DT}$, $\widetilde{n}_I$, $\widetilde{E}$ to zero using actuation through $\widetilde{P}_{aux}$, $\widetilde{S}$ *and* $\widetilde{S}_I$. All the states are assumed to be available for feedback from measurement or estimation.

TABLE I: ITER Machine Parameters [8]

Symbol	Quantity	Value
I	Plasma Current	22.0 MA
R	Major Radius	6.0 m
A	Minor Radius	2.15 m
B	Magnetic Field	4.85 T
κ_χ	Elongation at χ	2.2
K_α	Alpha particle confinement cte.	7
k_{DT}	DT particle confinement cte.	3
k_I	Impurity particle confinement cte.	10
β_{max}	Beta limit	2.5I/aB=5.3%
V	Plasma Volume	1100 m^3

IV. Controller Design

We start by looking for a control that stabilizes $\widetilde{E}$. We choose u such that

$$\frac{\overline{E}}{\tau_E} - \left[\left(\frac{n_{DT}}{2}\right)^2 \langle\sigma v\rangle Q_\alpha + P_{aux} - P_{rad}\right] = 0 \tag{17}$$

This means that we choose P_{aux} and n_I such that

$$\frac{\overline{E}}{\tau_E} = \left(\frac{n_{DT}}{2}\right)^2 \langle\sigma v\rangle Q_\alpha - A_b Z_{eff} n_e^2 \sqrt{T} + P_{aux} = P \quad (18)$$

From the equilibrium equation for the energy E, $0 = -\overline{E}/\overline{\tau}_E + \overline{P}$, and the expression (10) for the energy confinement time, we note that the solution for (18) is $P = \overline{P}$. Therefore, the control strategy will be to adjust P_{aux} and n_I, if necessary, to make *P* constant and equal to $\overline{P}$ satisfying (18) and reducing (16) to

$$\frac{d\widetilde{E}}{dt} = -\frac{\widetilde{E}}{\tau_E}$$

The subsystem $\widetilde{E}$ is exponential stable since $\tau_E > 0$. The controller that implements (18) is synthesized in two steps:

First Step: We compute

$$P_{aux} = \overline{P} - \left(\frac{n_{DT}}{2}\right)^2 \langle\sigma v\rangle Q_\alpha + A_b Z_{eff} n_e^2 \sqrt{T} \quad (19)$$

If $P_{aux} \geq 0$ then we keep this value for P_{aux} and let $S_I = 0$.

If $P_{aux} < 0$ then we take $P_{aux} = 0$ and go to the *Second Step*,

Second Step: Noting that n_e, Z_{eff} and *T* are functions of n_I, we follow a singular perturbation approach and we look for the least $n_I = n_I^* > 0$ such that

$$-\overline{P} + \left(\frac{n_{DT}}{2}\right)^2 \langle\sigma v\rangle Q_\alpha = A_b Z_{eff} n_e^2 \sqrt{T}$$

and use this value n_I^* as the reference to follow for the positive valued proportional controller:

$$\begin{aligned} S_I &= K_I (n_I^* - n_I) \quad & \forall n_I^* - n_I \geq 0 \\ S_I &= 0 & \forall n_I^* - n_I < 0 \end{aligned} \quad (20)$$

We note from (15) that $\widetilde{n}_I$ is input-state stable (ISS) ([10], section 5.3) with respect to S_I. This ensures that $\widetilde{n}_I$ will be bounded as long as S_I is bounded, and it will be exponentially stable once S_I becomes zero. After stabilizing $\widetilde{E}$ and $\widetilde{n}_I$ using P_{aux} and S_I as controllers, we must focus on (13) and (14) to achieve stability for $\widetilde{n}_{DT}$ and $\widetilde{n}_\alpha$. Choosing

$$\widetilde{S} = 2\left(\frac{\widetilde{n}_{DT}}{2}\right)^2 \langle\sigma v\rangle - u_{DT} \quad (21)$$

we can reduce (14) to

$$\frac{d\widetilde{n}_{DT}}{dt} = -\left[\frac{1}{\tau_{DT}} + \overline{n}_{DT}\langle\sigma v\rangle\right]\widetilde{n}_{DT}$$

Since $1/\tau_{DT} + \overline{n}_{DT}\langle\sigma v\rangle$ is positive, the subsystem $\widetilde{n}_{DT}$ is exponential stable.

In order to finish our stability analysis we examine the expression (13) for $\widetilde{n}_\alpha$. We note that $\widetilde{n}_\alpha$ is ISS with respect to $\widetilde{n}_{DT}$ and u_α. Therefore, since $\widetilde{n}_{DT}$ is bounded (because it is exponentially stable), and u_α is bounded (because $\widetilde{E}$ is exponentially stable and $\langle\sigma v\rangle$ is a bounded function), $\widetilde{n}_\alpha$ will be bounded for all time. In addition, once *E* converges to $\overline{E}$ ($\widetilde{E} \to 0$) and n_{DT} converges to $\overline{n}_{DT}$ ($\widetilde{n}_{DT} \to 0$) this equation reduces to

$$\frac{d\widetilde{n}_\alpha}{dt} = -\frac{\widetilde{n}_\alpha}{\overline{\tau}_\alpha} + u_\alpha^*, \qquad u_\alpha^* = \frac{\overline{n}_\alpha}{\overline{\tau}_\alpha} + \left(\frac{\overline{n}_{DT}}{2}\right)^2 \langle\sigma v\rangle \quad (22)$$

The function $\langle\sigma v\rangle$ is a function of $T = 2E/3(2n_{DT} + 3n_\alpha)$, once $n_I = \widetilde{n}_I$ converges to zero, and has a positive derivative in the region of interest. Consequently u_α^* has the same sign as $-\widetilde{n}_\alpha/\overline{\tau}_\alpha$ and vanishes when $\widetilde{n}_\alpha$ vanishes ($\langle\sigma v\rangle = \langle\overline{\sigma v}\rangle$). This allows us to conclude exponential stability for $\widetilde{n}_\alpha$.

V. Simulation Results

The objective of the controller is to keep the plasma at a desired equilibrium or operating point. The controller must be able to reject perturbations in initial conditions, forcing the plasma back to the equilibrium. For all the simulations presented here we have used impurities with $Z_I = 8$. This relatively low *Z* and the fusion reactor temperature justify the absence of the term corresponding to the line radiation due to impurities in the energy balance equation of our model [9]. In addition, a controller gain $K_I = 0.05$ and a scale factor *f*=0.85 for the energy confinement time (10) have been used. It should be noted that our controller does not depend on k_I and consequently it tolerates any size of uncertainty in this parameter. Therefore the choice of $k_I = 10$ can be considered completely arbitrary and with the only purpose of the simulation.

The controller designed shows capability of rejecting different types of large perturbations in initial conditions. Figure 1 compares its performance with other two controllers synthesized by linear pole placement [4] and linear robust [5] techniques which use the same dynamical model presented here. This study is carried out generating initial perturbations around the equilibrium ($\overline{T}$=8.28 *KeV*, $\overline{n}_e$=9.80 10^{19} m^{-3}, $\overline{f}_\alpha$=6.41%, $\overline{\beta}$=2.65%, $\overline{n}_\alpha$=6.28 10^{18} m^{-3}, $\overline{n}_{DT}$=8.55 10^{19} m^{-3}, $\overline{E}$=3.78 10^5 Jm^{-3}, $\overline{P}_{aux}$=0 Wm^{-3}, $\overline{S}$=4.04 10^{18} m^{-3} sec^{-1}.) for *T* and n_e, keeping the alpha-particle fraction $f_\alpha = n_\alpha/n_e$ equal to that of the equilibrium. While the boundaries shown for the linear controllers are absolute, for the nonlinear controller they only indicate the limits within which we performed our tests.

The robustness of our controller was also studied against those of the linear controllers. Figure 2 shows the regions of stability against uncertainty in the parameter k_α whose nominal value is equal to 7 when the system suffers perturbations in the initial temperature. Again, the region shown for the nonlinear controller is not a limit. With the sole objective to show its performance we tested it against uncertainties up to 400 % and perturbations for initial *T* between -90 % and 100 %.

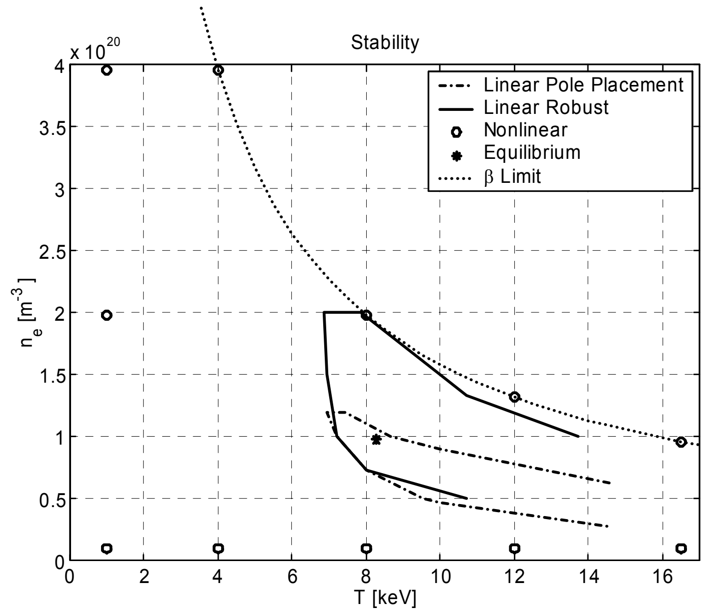

Fig. 1. Stability Comparison.

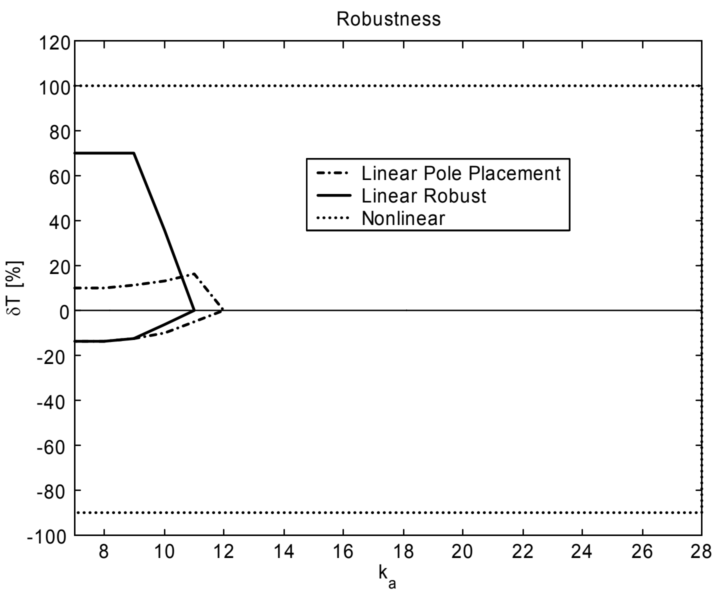

Fig. 2. Robustness Comparison.

VI. CONCLUSIONS AND FUTURE WORK

This new approach to the problem of burn control allows us to deal with perturbations in initial conditions that were unmanageable until now. On the other hand, the multi-input nature of the controller allows it to reject large perturbations in initial conditions leading to both thermal excursion and quenching. In addition, the effectiveness of the controller does not depend on whether the operating point is an ignition or a subignition point.

Since the nonlinear controller depends parametrically on the equilibrium point, it can drive the system from one equilibrium point to another allowing in this way the change of power, other plasma parameters and ignition conditions. No scheduled controllers are necessary and the same control law is valid for every equilibrium point.

Simulation results show good robustness properties against uncertainties in the confinement times. The control laws (19), (20) and (21) are only functions of k_{DT} and even this dependence in (21) can be avoided with a slight modification in the design that is not presented in this work. The boundedness of the system solutions is achieved for any kind and size of perturbation in initial conditions regardless of the size and nature of the uncertainty. The controller is always robust against uncertainties in k_I, is always able to drive $E \rightarrow \overline{E}$ regardless of the uncertainty type and, in addition, is able to drive $n_{DT} \rightarrow \overline{n}_{DT}$ when there is no uncertainty in k_{DT}. In order to drive the system to the equilibrium point corresponding to the actual values of the confinement times, and to avoid spending control effort on handling the uncertainties in an unstructured (non-parametric) manner, a nonlinear adaptive control law should be synthesized.

It must be noted that this approach can be extended to the use of any other energy confinement time scaling based on the net heating power.

One possible extension of this work involves developing a more accurate model which includes radiation terms for higher Z impurities and other phenomena like injected fuel diffusion. Finally, in order to approach a more relevant problem in the fusion context as the control of the kinetic profiles, a one-dimensional dynamic model should be introduced and a nonlinear distributed controller should be synthesized.

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